

Specifications and Calculations for GuQin Construction

I. Defining chi and cun

$$\text{cun} \equiv \frac{11}{9} \cdot 2.54\text{cm} \quad \text{chi} \equiv 10\text{cun} \quad \text{ORIGIN} \equiv 1$$

II. Determining the distance to each Line of Reference from the Baseline

Surface Dimensions:

12 Intervals between 13 Lines of Reference

$$\text{interval} := 1 .. 12 \quad \text{lines} := 1 .. 13 \quad \text{successive_lines} := 2 .. 13$$

These are the distance from each Line of Reference to the next, between lines 1 through 13.

Spacing_{interval} :=

Line 01 Baseline	2cun
Line 02 Forehead	0.4cun
Line 03 Receiving Dew	0.3cun
Line 04 Bridge	0.8cun
Line 05 Beginning of Nape	3.2cun
Line 06 Center of Neck	3.2cun
Line 07 Shoulder	1chi + 4.4cun
Line 08 Beginning of Waist	3.6cun
Line 09 Center of Waist	3.6cun
Line 10 End of Waist	6.2cun
Line 11 Cap	1cun
Line 12 Tail-end	0.3cun
Line 13 Mouth of Nut	0.3cun

Calculation of Cumulative Distances from Baseline to Lines of Reference

$$\text{Distance}(\text{begin}, \text{end}) := \sum_{i = \text{begin}}^{\text{end}-1} \text{Spacing}_i$$

$$\text{Dist}_{\text{successive_lines}} := \text{Distance}(1, \text{successive_lines})$$

	$\left(\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \end{array} \right)$	$\text{lines} =$	$\text{Dist} =$	$\left(\begin{array}{c} 0 \\ 2 \\ 2.4 \\ 2.7 \\ 3.5 \\ 6.7 \\ 9.9 \\ 24.3 \\ 27.9 \\ 31.5 \\ 37.7 \\ 38.7 \\ 39 \end{array} \right)$	cun	$\text{Dist} =$	$\left(\begin{array}{c} 0.00 \\ 2.44 \\ 2.93 \\ 3.30 \\ 4.28 \\ 8.19 \\ 12.10 \\ 29.70 \\ 34.10 \\ 38.50 \\ 46.08 \\ 47.30 \\ 47.67 \end{array} \right)$	in	$\text{Dist} =$	$\left(\begin{array}{c} 0 \\ 6.209 \\ 7.451 \\ 8.382 \\ 10.866 \\ 20.8 \\ 30.734 \\ 75.438 \\ 86.614 \\ 97.79 \\ 117.038 \\ 120.142 \\ 121.073 \end{array} \right)$	cm
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Distances
between
Baseline to
Lines of
Reference in
cun , inches,
and
centimeters

III. Calculating the width of the GuQin at each Line of Reference

Width of GuQin from Axis Line to Edge with respect to each Line of Reference

	$\text{Width}_1 := 5.6\text{cun}$
	$\text{Width}_5 := 6\text{cun}$
Line 01 Baseline Line 02 Forehead Line 03 Receiving Dew Line 04 Bridge Line 05 Beginning of Nape Line 06 Center of Neck Line 07 Shoulder Line 08 Beginning of Waist Line 09 Center of Waist Line 10 End of Waist	$\text{Width}_2 := 2 \cdot \left[\frac{\left(\frac{\text{Width}_5}{2} - \frac{\text{Width}_1}{2} \right)}{\text{Distance}(1,5)} \cdot \text{Distance}(1,2) + \frac{\text{Width}_1}{2} \right] + 1 \cdot \text{cun}$
Line 11 Cap Line 12 Tail-end Line 13 Mouth of Nut	$\text{Width}_4 := 2 \cdot \left[\frac{\left(\frac{\text{Width}_5}{2} - \frac{\text{Width}_1}{2} \right)}{\text{Distance}(1,5)} \cdot \text{Distance}(1,4) + \frac{\text{Width}_1}{2} \right] + 1 \cdot \text{cun}$
	$\text{Width}_6 := 5.6\text{cun}$
	$\text{Width}_7 := 7 \cdot \text{cun}$
	$\text{Width}_{11} := 5\text{cun}$

$$\text{Width}_9 := 2 \cdot \left[\frac{\left(\frac{\text{Width}_{11}}{2} - \frac{\text{Width}_7}{2} \right)}{\text{Distance}(7, 11)} \cdot \text{Distance}(7, 9) + \frac{\text{Width}_7}{2} \right] - 0.3 \text{ cun}$$

$$\text{Width}_{12} := 2 \cdot \left[\frac{\left(\frac{\text{Width}_{11}}{2} - \frac{\text{Width}_7}{2} \right)}{\text{Distance}(7, 11)} \cdot \text{Distance}(7, 12) + \frac{\text{Width}_7}{2} \right]$$

$$\text{Width}_{13} := 1.2 \text{ cun}$$

$$\text{Half_Width} := \frac{\text{Width}}{2}$$

$$\text{Half_Width}_1 = 2.8 \text{ cun}$$

$$\text{Half_Width}_2 = 3.414 \text{ cun}$$

$$\text{Half_Width}_4 = 3.454 \text{ cun}$$

$$\text{Half_Width}_5 = 3 \text{ cun}$$

$$\text{Half_Width}_6 = 2.8 \text{ cun}$$

$$\text{Half_Width}_7 = 3.5 \text{ cun}$$

$$\text{Half_Width}_9 = 2.703 \text{ cun}$$

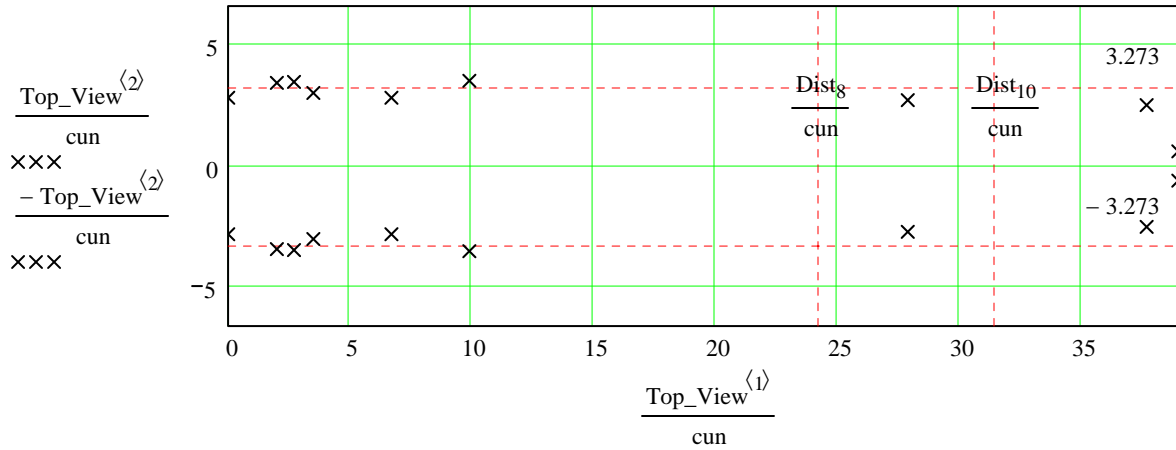
$$\text{Half_Width}_{11} = 2.5 \text{ cun}$$

$$\text{Half_Width}_{12} = 2.464 \text{ cun}$$

$$\text{Half_Width}_{13} = 0.6 \text{ cun}$$

$$\text{Top_View} := \begin{pmatrix} \text{Dist}_1 & \text{Half_Width}_1 \\ \text{Dist}_2 & \text{Half_Width}_2 \\ \text{Dist}_4 & \text{Half_Width}_4 \\ \text{Dist}_5 & \text{Half_Width}_5 \\ \text{Dist}_6 & \text{Half_Width}_6 \\ \text{Dist}_7 & \text{Half_Width}_7 \\ \text{Dist}_9 & \text{Half_Width}_9 \\ \text{Dist}_{11} & \text{Half_Width}_{11} \\ \text{Dist}_{13} & \text{Half_Width}_{13} \end{pmatrix}$$

$$\text{Top_View} = \begin{pmatrix} 0 & 8.692 \\ 6.209 & 10.599 \\ 8.382 & 10.724 \\ 10.866 & 9.313 \\ 20.8 & 8.692 \\ 30.734 & 10.866 \\ 86.614 & 8.39 \\ 117.038 & 7.761 \\ 121.073 & 1.863 \end{pmatrix} \text{ cm}$$



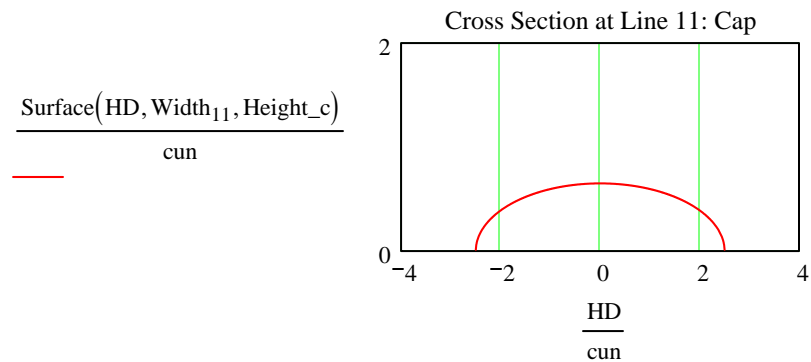
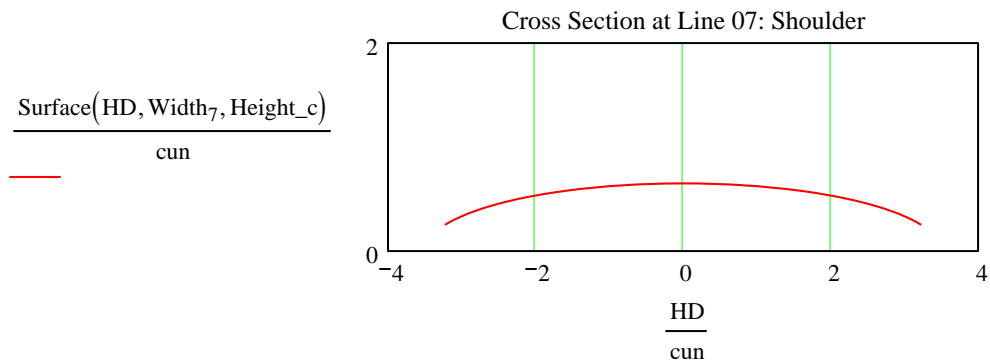
IV. Developing the foundation for curvature calculation

IV.A. Setting the Reference curvatures at the Shoulder and Nut

Calculations for Surface Contours

$$\text{Height}_c := 0.65\text{cun}$$

$$\text{Surface}(\text{HD}, w, h) := \sqrt[3]{1 - \frac{\text{HD}^2}{\left(\frac{w}{2}\right)^2}} \cdot h^2$$



Calculating Longitudinal Slope Along Each String

$$\text{String} := 1 .. 7$$

$$\text{Shoulder_String_Width} := 2 \cdot \left[\frac{(0.55\text{cun} - 2\text{cun})}{\text{Distance}(4, 12)} \cdot \text{Distance}(4, 7) + 2\text{cun} \right]$$

$$\text{Bridge_String_Width} := 4\text{cun}$$

$$\text{Shoulder_String_Width} = 3.42\text{cun}$$

$$\text{Nut_String_Width} := 1.1\text{cun}$$

$$\text{Shoulder_String_X_Position}_{\text{String}} := (\text{String} - 4) \cdot \frac{\text{Shoulder_String_Width}}{6}$$

$$\text{Nut_String_X_Position}_{\text{String}} := (\text{String} - 4) \cdot \frac{\text{Nut_String_Width}}{6}$$

$$\text{Shoulder_String_X_Position} = \begin{pmatrix} -1.71 \\ -1.14 \\ -0.57 \\ 0 \\ 0.57 \\ 1.14 \\ 1.71 \end{pmatrix} \text{cun} \quad \text{Nut_String_X_Position} = \begin{pmatrix} -0.55 \\ -0.367 \\ -0.183 \\ 0 \\ 0.183 \\ 0.367 \\ 0.55 \end{pmatrix} \text{cun}$$

IV.B. Extracting points of the curvature at the Shoulder and Nut for the purpose of interpolation

$$N_X1 := \frac{-(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 8 - 0.55\text{cun}$$

$$N_X2 := \frac{-(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 7 - 0.55\text{cun}$$

$$N_X3 := \frac{-(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 6 - 0.55\text{cun}$$

$$N_X4 := \frac{-(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 5 - 0.55\text{cun}$$

$$N_X5 := \frac{-(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 4 - 0.55\text{cun}$$

$$N_X6 := \frac{-(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 3 - 0.55\text{cun}$$

$$N_X7 := \frac{-(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 2 - 0.55\text{cun}$$

$$N_X8 := \frac{-(\text{Half_Width}_{12} - 0.55\text{cun})}{8} - 0.55\text{cun}$$

$$N_X9 := -(0.55\text{cun})$$

$$N_X10 := \text{Nut_String_X_Position}_2$$

$$N_X11 := \text{Nut_String_X_Position}_3$$

$$N_X12 := \text{Nut_String_X_Position}_4$$

$$N_X13 := \text{Nut_String_X_Position}_5$$

$$N_X14 := \text{Nut_String_X_Position}_6$$

$$N_X15 := 0.55\text{cun}$$

$$N_X16 := \frac{\text{Half_Width}_{12} - 0.55\text{cun}}{8} + 0.55\text{cun}$$

$$N_X17 := \frac{(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 2 + 0.55\text{cun}$$

$$N_X18 := \frac{(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 3 + 0.55\text{cun}$$

$$N_X19 := \frac{(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 4 + 0.55\text{cun}$$

$$N_X20 := \frac{(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 5 + 0.55\text{cun}$$

$$N_X21 := \frac{(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 6 + 0.55\text{cun}$$

$$N_X22 := \frac{(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 7 + 0.55\text{cun}$$

$$N_X23 := \frac{(\text{Half_Width}_{12} - 0.55\text{cun})}{8} \cdot 8 + 0.55\text{cun}$$

$$N_XY := \left(\begin{array}{l} N_X1 \text{ Surface}(N_X1, \text{Width}_{12}, \text{Height}_c) \\ N_X2 \text{ Surface}(N_X2, \text{Width}_{12}, \text{Height}_c) \\ N_X3 \text{ Surface}(N_X3, \text{Width}_{12}, \text{Height}_c) \\ N_X4 \text{ Surface}(N_X4, \text{Width}_{12}, \text{Height}_c) \\ N_X5 \text{ Surface}(N_X5, \text{Width}_{12}, \text{Height}_c) \\ N_X6 \text{ Surface}(N_X6, \text{Width}_{12}, \text{Height}_c) \\ N_X7 \text{ Surface}(N_X7, \text{Width}_{12}, \text{Height}_c) \\ N_X8 \text{ Surface}(N_X8, \text{Width}_{12}, \text{Height}_c) \\ N_X9 \text{ Surface}(N_X9, \text{Width}_{12}, \text{Height}_c) \\ N_X10 \text{ Surface}(N_X10, \text{Width}_{12}, \text{Height}_c) \\ N_X11 \text{ Surface}(N_X11, \text{Width}_{12}, \text{Height}_c) \\ N_X12 \text{ Surface}(N_X12, \text{Width}_{12}, \text{Height}_c) \\ N_X13 \text{ Surface}(N_X13, \text{Width}_{12}, \text{Height}_c) \\ N_X14 \text{ Surface}(N_X14, \text{Width}_{12}, \text{Height}_c) \\ N_X15 \text{ Surface}(N_X15, \text{Width}_{12}, \text{Height}_c) \\ N_X16 \text{ Surface}(N_X16, \text{Width}_{12}, \text{Height}_c) \\ N_X17 \text{ Surface}(N_X17, \text{Width}_{12}, \text{Height}_c) \\ N_X18 \text{ Surface}(N_X18, \text{Width}_{12}, \text{Height}_c) \\ N_X19 \text{ Surface}(N_X19, \text{Width}_{12}, \text{Height}_c) \\ N_X20 \text{ Surface}(N_X20, \text{Width}_{12}, \text{Height}_c) \\ N_X21 \text{ Surface}(N_X21, \text{Width}_{12}, \text{Height}_c) \\ N_X22 \text{ Surface}(N_X22, \text{Width}_{12}, \text{Height}_c) \\ N_X23 \text{ Surface}(N_X23, \text{Width}_{12}, \text{Height}_c) \end{array} \right) \quad N_XY = \left(\begin{array}{l} -2.464 \quad 0 \\ -2.225 \quad 0.279 \\ -1.986 \quad 0.385 \\ -1.746 \quad 0.459 \\ -1.507 \quad 0.514 \\ -1.268 \quad 0.557 \\ -1.029 \quad 0.591 \\ -0.789 \quad 0.616 \\ -0.55 \quad 0.634 \\ -0.367 \quad 0.643 \\ -0.183 \quad 0.648 \\ 0 \quad 0.65 \\ 0.183 \quad 0.648 \\ 0.367 \quad 0.643 \\ 0.55 \quad 0.634 \\ 0.789 \quad 0.616 \\ 1.029 \quad 0.591 \\ 1.268 \quad 0.557 \\ 1.507 \quad 0.514 \\ 1.746 \quad 0.459 \\ 1.986 \quad 0.385 \\ 2.225 \quad 0.279 \\ 2.464 \quad 0 \end{array} \right) \text{cun}$$

$$S_X1 := \frac{-(\text{Half_Width}_7 + \text{Shoulder_String_X_Position}_1)}{8} \cdot 8 + \text{Shoulder_String_X_Position}_1$$

$$S_X2 := \frac{-(\text{Half_Width}_7 + \text{Shoulder_String_X_Position}_1)}{8} \cdot 7 + \text{Shoulder_String_X_Position}_1$$

$$S_X3 := \frac{-(\text{Half_Width}_7 + \text{Shoulder_String_X_Position}_1)}{8} \cdot 6 + \text{Shoulder_String_X_Position}_1$$

$$S_X4 := \frac{-(\text{Half_Width}_7 + \text{Shoulder_String_X_Position}_1)}{8} \cdot 5 + \text{Shoulder_String_X_Position}_1$$

$$S_X5 := \frac{-(\text{Half_Width}_7 + \text{Shoulder_String_X_Position}_1)}{8} \cdot 4 + \text{Shoulder_String_X_Position}_1$$

$$S_X6 := \frac{-(\text{Half_Width}_7 + \text{Shoulder_String_X_Position}_1)}{8} \cdot 3 + \text{Shoulder_String_X_Position}_1$$

$$S_X7 := \frac{-(\text{Half_Width}_7 + \text{Shoulder_String_X_Position}_1)}{8} \cdot 2 + \text{Shoulder_String_X_Position}_1$$

$$S_X8 := \frac{-(\text{Half_Width}_7 + \text{Shoulder_String_X_Position}_1)}{8} + \text{Shoulder_String_X_Position}_1$$

$$S_X9 := \text{Shoulder_String_X_Position}_1$$

$$S_X10 := \text{Shoulder_String_X_Position}_2$$

$$S_X11 := \text{Shoulder_String_X_Position}_3$$

$$S_X12 := \text{Shoulder_String_X_Position}_4$$

$$S_X13 := \text{Shoulder_String_X_Position}_5$$

$$S_X14 := \text{Shoulder_String_X_Position}_6$$

$$S_X15 := \text{Shoulder_String_X_Position}_7$$

$$S_X16 := \frac{(\text{Half_Width}_7 - \text{Shoulder_String_X_Position}_7)}{8} + \text{Shoulder_String_X_Position}_7$$

$$S_X17 := \frac{(\text{Half_Width}_7 - \text{Shoulder_String_X_Position}_7)}{8} \cdot 2 + \text{Shoulder_String_X_Position}_7$$

$$S_X18 := \frac{(\text{Half_Width}_7 - \text{Shoulder_String_X_Position}_7)}{8} \cdot 3 + \text{Shoulder_String_X_Position}_7$$

$$S_X19 := \frac{(\text{Half_Width}_7 - \text{Shoulder_String_X_Position}_7)}{8} \cdot 4 + \text{Shoulder_String_X_Position}_7$$

$$S_{X20} := \frac{(\text{Half_Width}_7 - \text{Shoulder_String_X_Position}_7)}{8} \cdot 5 + \text{Shoulder_String_X_Position}_7$$

$$S_{X21} := \frac{(\text{Half_Width}_7 - \text{Shoulder_String_X_Position}_7)}{8} \cdot 6 + \text{Shoulder_String_X_Position}_7$$

$$S_{X22} := \frac{(\text{Half_Width}_7 - \text{Shoulder_String_X_Position}_7)}{8} \cdot 7 + \text{Shoulder_String_X_Position}_7$$

$$S_{X23} := \frac{(\text{Half_Width}_7 - \text{Shoulder_String_X_Position}_7)}{8} \cdot 8 + \text{Shoulder_String_X_Position}_7$$

$$S_{XY} := \left(\begin{array}{l} S_{X1} \text{ Surface}(S_{X1}, \text{Width}_7, \text{Height}_c) \\ S_{X2} \text{ Surface}(S_{X2}, \text{Width}_7, \text{Height}_c) \\ S_{X3} \text{ Surface}(S_{X3}, \text{Width}_7, \text{Height}_c) \\ S_{X4} \text{ Surface}(S_{X4}, \text{Width}_7, \text{Height}_c) \\ S_{X5} \text{ Surface}(S_{X5}, \text{Width}_7, \text{Height}_c) \\ S_{X6} \text{ Surface}(S_{X6}, \text{Width}_7, \text{Height}_c) \\ S_{X7} \text{ Surface}(S_{X7}, \text{Width}_7, \text{Height}_c) \\ S_{X8} \text{ Surface}(S_{X8}, \text{Width}_7, \text{Height}_c) \\ S_{X9} \text{ Surface}(S_{X9}, \text{Width}_7, \text{Height}_c) \\ S_{X10} \text{ Surface}(S_{X10}, \text{Width}_7, \text{Height}_c) \\ S_{X11} \text{ Surface}(S_{X11}, \text{Width}_7, \text{Height}_c) \\ S_{X12} \text{ Surface}(S_{X12}, \text{Width}_7, \text{Height}_c) \\ S_{X13} \text{ Surface}(S_{X13}, \text{Width}_7, \text{Height}_c) \\ S_{X14} \text{ Surface}(S_{X14}, \text{Width}_7, \text{Height}_c) \\ S_{X15} \text{ Surface}(S_{X15}, \text{Width}_7, \text{Height}_c) \\ S_{X16} \text{ Surface}(S_{X16}, \text{Width}_7, \text{Height}_c) \\ S_{X17} \text{ Surface}(S_{X17}, \text{Width}_7, \text{Height}_c) \\ S_{X18} \text{ Surface}(S_{X18}, \text{Width}_7, \text{Height}_c) \\ S_{X19} \text{ Surface}(S_{X19}, \text{Width}_7, \text{Height}_c) \\ S_{X20} \text{ Surface}(S_{X20}, \text{Width}_7, \text{Height}_c) \\ S_{X21} \text{ Surface}(S_{X21}, \text{Width}_7, \text{Height}_c) \\ S_{X22} \text{ Surface}(S_{X22}, \text{Width}_7, \text{Height}_c) \\ S_{X23} \text{ Surface}(S_{X23}, \text{Width}_7, \text{Height}_c) \end{array} \right) \quad S_{XY} = \begin{pmatrix} -3.5 & 0 \\ -3.276 & 0.229 \\ -3.052 & 0.318 \\ -2.829 & 0.383 \\ -2.605 & 0.434 \\ -2.381 & 0.476 \\ -2.157 & 0.512 \\ -1.934 & 0.542 \\ -1.71 & 0.567 \\ -1.14 & 0.615 \\ -0.57 & 0.641 \\ 0 & 0.65 \\ 0.57 & 0.641 \\ 1.14 & 0.615 \\ 1.71 & 0.567 \\ 1.934 & 0.542 \\ 2.157 & 0.512 \\ 2.381 & 0.476 \\ 2.605 & 0.434 \\ 2.829 & 0.383 \\ 3.052 & 0.318 \\ 3.276 & 0.229 \\ 3.5 & 0 \end{pmatrix} \text{ cun}$$

V. Calculating Bridge geometry

V.A. Interpolating data points at the Bridge, using the Reference curvatures at the Shoulder and Nut

$$B_XY := \frac{(S_XY - N_XY)}{\text{Distance}(7, 12)} \cdot \text{Distance}(4, 12) + N_XY$$

$$B_X := B_XY^{(1)}$$

$$B_Y := B_XY^{(2)}$$

$$B_XY = \begin{pmatrix} -3.759 & 0 \\ -3.539 & 0.216 \\ -3.319 & 0.301 \\ -3.099 & 0.364 \\ -2.879 & 0.414 \\ -2.66 & 0.456 \\ -2.44 & 0.492 \\ -2.22 & 0.523 \\ -2 & 0.551 \\ -1.333 & 0.608 \\ -0.667 & 0.64 \\ 0 & 0.65 \\ 0.667 & 0.64 \\ 1.333 & 0.608 \\ 2 & 0.551 \\ 2.22 & 0.523 \\ 2.44 & 0.492 \\ 2.66 & 0.456 \\ 2.879 & 0.414 \\ 3.099 & 0.364 \\ 3.319 & 0.301 \\ 3.539 & 0.216 \\ 3.759 & 0 \end{pmatrix} \text{ cun}$$

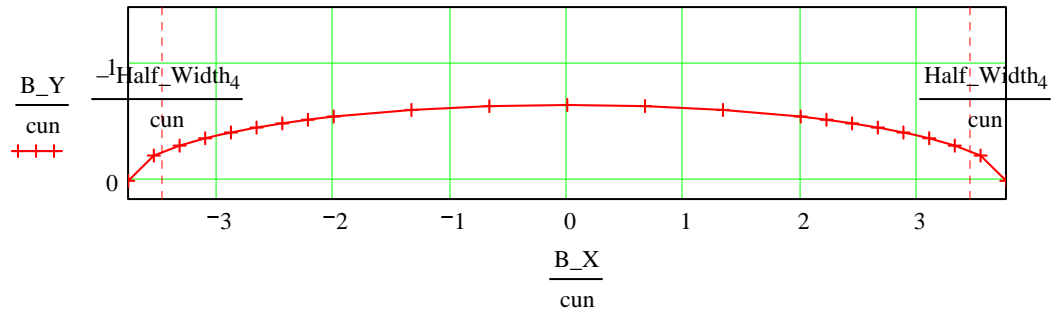
V.B. Deriving the Function $B_Y_interp(x)$ describing the curvature of the interpolated data points

$$B_S := \text{pspline}(B_X, B_Y)$$

Spline vector constant

$$B_Y_interp(X) := \text{interp}(B_S, B_X, B_Y, X)$$

Curvature at bridge from interpolation of the face

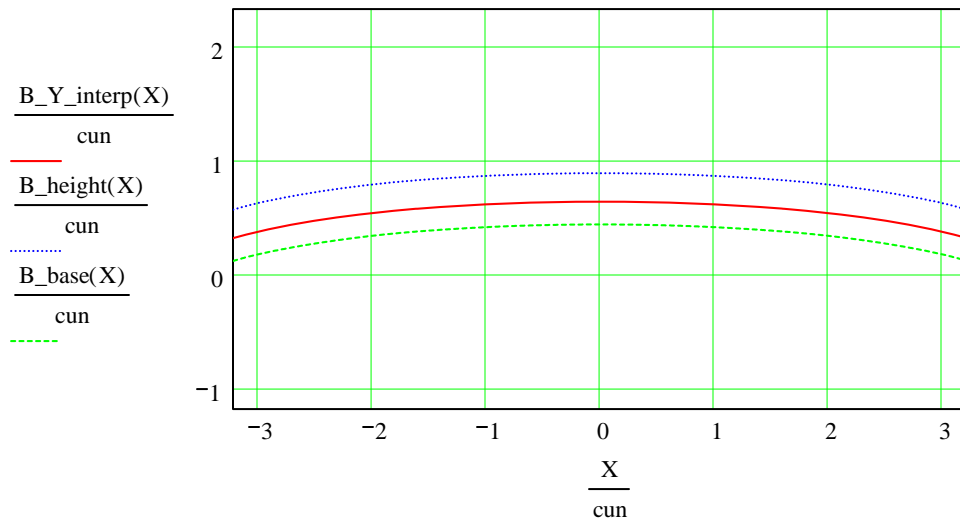


V.C. Deriving the function $B_height(x)$ describing the curvature of the top of the Bridge

$$B_height(X) := B_Y_interp(X) + 0.25cun$$

V.D. Deriving the function $B_base(x)$ describing the curvature of the bottom of the Bridge

$$B_base(X) := B_Y_interp(X) - 0.2cun$$



V.E. Ensuring equal spacing of Strings at the Bridge

V.E.1. Calculation of String spacing measured along the curvature of the top of the bridge

B_sw = The length of curvature along the top of the bridge, from string 1 to string 7.

B_sp = The distance of each String Position measured along the curvature of the top of the bridge.

$$B_{sw} := \int_{\frac{-\text{Bridge_String_Width}}{2}}^{\frac{\text{Bridge_String_Width}}{2}} \sqrt{1 + \left(\frac{d}{dX} B_{\text{height}}(X)\right)^2} dX$$

$$B_{sw} = 4.007 \text{ cun}$$

$$B_{sp_String} := (\text{String} - 4) \cdot \frac{B_{sw}}{6}$$

$$B_{sp} = \begin{pmatrix} -2.003 \\ -1.336 \\ -0.668 \\ 0 \\ 0.668 \\ 1.336 \\ 2.003 \end{pmatrix} \text{ cun}$$

X Positon of Each String at Bridge Top

B_sX_bt = The X coordinate of each string while seated at bridge top, measured horizontally from the center axis of the GuQin , while B_sY_bt = the Y coordinate.

For Strings 1 and 7

$$x_dist := 2\text{cun}$$

Given

$$B_{sp_7} = \int_{0\text{cun}}^{x_dist} \sqrt{1 + \left(\frac{d}{dX} B_{\text{height}}(X)\right)^2} dX$$

$$B_{sX_bt_7} := \text{Find}(x_dist)$$

$$B_{sX_bt_1} := -B_{sX_bt_7}$$

For Strings 2 and 6

$$x_dist := 1.3\text{cun}$$

Given

$$B_sp_6 = \int_{0\text{cun}}^{x_dist} \sqrt{1 + \left(\frac{d}{dX} B_height(X)\right)^2} dX$$

$$B_sX_bt_6 := \text{Find}(x_dist)$$

$$B_sX_bt_2 := -B_sX_bt_6$$

For Strings 3 and 5

$$x_dist := 0.7\text{cun}$$

Given

$$B_sp_5 = \int_{0\text{cun}}^{x_dist} \sqrt{1 + \left(\frac{d}{dX} B_height(X)\right)^2} dX$$

$$B_sX_bt_5 := \text{Find}(x_dist)$$

$$B_sX_bt_3 := -B_sX_bt_5$$

$$B_sY_bt_{\text{String}} := B_height(B_sX_bt_{\text{String}})$$

$$B_S_bt^{(1)} := B_sX_bt$$

$$B_S_bt^{(2)} := B_sY_bt$$

These are the X,Y coordinates of each string at the height of the bridge.

$$B_S_bt = \begin{pmatrix} -2 & 0.801 \\ -1.335 & 0.857 \\ -0.668 & 0.89 \\ 0 & 0.9 \\ 0.668 & 0.89 \\ 1.335 & 0.857 \\ 2 & 0.801 \end{pmatrix} \text{cun}$$

These are the distance from the center axis line along the top of the bridge for each string.

$$B_{sp} = \begin{pmatrix} -2.003 \\ -1.336 \\ -0.668 \\ 0 \\ 0.668 \\ 1.336 \\ 2.003 \end{pmatrix} \text{ cun}$$

V.E.2. Calculation of Tuning Hole positioning

V.E.2.a. Finding the XY position of each Tuning Hole

Calculation of the Slope of the Bridge at Each String Position

$$xx := B_{sX_bt_1}$$

$$\text{Slope}_1 := \frac{d}{dxx} B_{\text{height}}(xx)$$

$$xx := B_{sX_bt_2}$$

$$\text{Slope}_2 := \frac{d}{dxx} B_{\text{height}}(xx)$$

$$xx := B_{sX_bt_3}$$

$$\text{Slope}_3 := \frac{d}{dxx} B_{\text{height}}(xx)$$

$$xx := B_{sX_bt_4}$$

$$\text{Slope}_4 := \frac{d}{dxx} B_{\text{height}}(xx)$$

$$\text{Slope}_7 := -\text{Slope}_1$$

$$\text{Slope}_6 := -\text{Slope}_2$$

$$\text{Slope}_5 := -\text{Slope}_3$$

iSlope = inverse of Slope. This depicts the slope of the tuning cords, running perpendicular to the curvature of the bridge.

$$i\text{Slope}_1 := \frac{-1}{\text{Slope}_1} \quad i\text{Slope}_5 := \frac{-1}{\text{Slope}_5}$$

$$iSlope_2 := \frac{-1}{Slope_2} \quad iSlope_6 := \frac{-1}{Slope_6} \quad iSlope_4 := \infty$$

$$iSlope_3 := \frac{-1}{Slope_3} \quad iSlope_7 := \frac{-1}{Slope_7}$$

$$Slope = \begin{pmatrix} 0.114 \\ 0.064 \\ 0.032 \\ 0 \\ -0.032 \\ -0.064 \\ -0.114 \end{pmatrix} \quad iSlope = \begin{pmatrix} -8.803 \\ -15.646 \\ -31.349 \\ 1 \times 10^{307} \\ 31.349 \\ 15.646 \\ 8.803 \end{pmatrix}$$

Finding the X,Y coordinates of the intersection between the Tuning Cords and the face of the GuQin , after adjusting for the 2/10 cun lowering of the face.

For Strings 1 and 7

$$x := 2cun \quad y := 0.3cun$$

$$i := 1..2$$

Given

$$y = B_base(x)$$

$$y = iSlope_7 \cdot x + (B_sY_bt_7 - iSlope_7 \cdot B_sX_bt_7)$$

$$B_S_f7 := \text{Find}(x, y)$$

$$B_S_f1_i :=$$

$-B_S_f7_1$
$B_S_f7_2$

For Strings 2 and 6

Given

$$y = B_base(x)$$

$$y = iSlope_6 \cdot x + (B_sY_bt_6 - iSlope_6 \cdot B_sX_bt_6)$$

$$B_S_f6 := \text{Find}(x, y)$$

$$B_S_f2_1 :=$$

$-B_S_f6_1$
$B_S_f6_2$

For Strings 3 and 5

Given

$$y = B_base(x)$$

$$y = iSlope_5 \cdot x + (B_sY_bt_5 - iSlope_5 \cdot B_sX_bt_5)$$

$$B_S_f5 := \text{Find}(x, y)$$

$$B_S_f3_1 :=$$

$-B_S_f5_1$
$B_S_f5_2$

For String 4

$$B_S_f4_1 :=$$

0cun
$B_base(0cun)$

The X,Y coordinates of the tuning cord and face intersection definition:

$$tB_S_f^{(1)} := B_S_f1$$

$$tB_S_f^{(5)} := B_S_f5$$

$$tB_S_f^{(2)} := B_S_f2$$

$$tB_S_f^{(6)} := B_S_f6$$

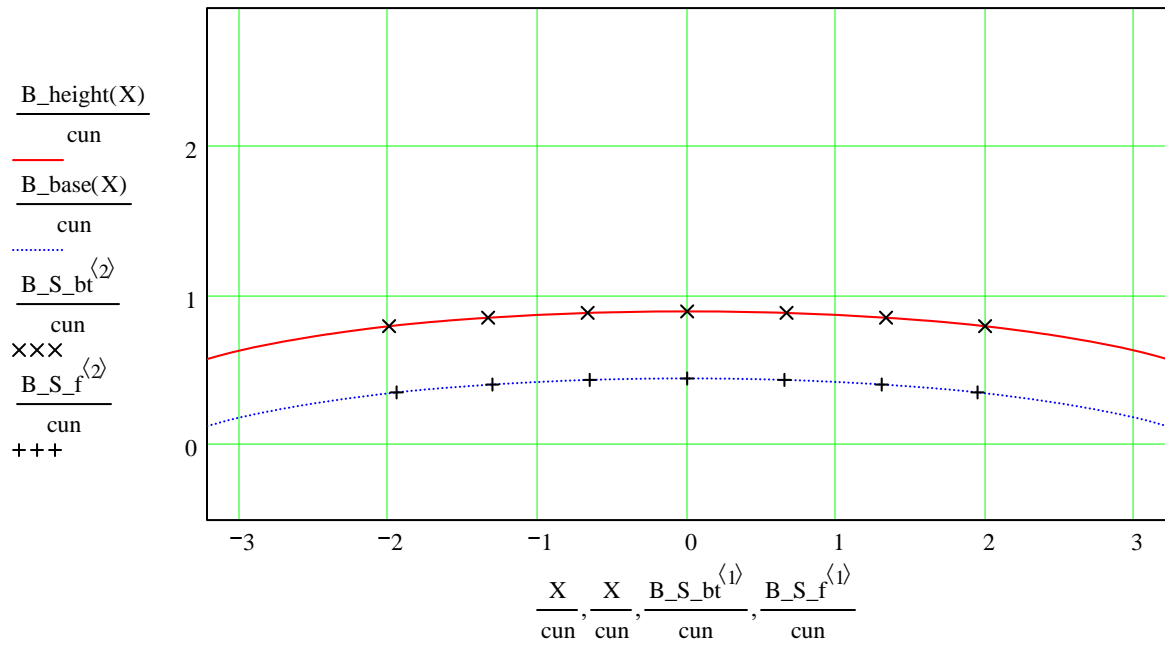
$$tB_S_f^{(3)} := B_S_f3$$

$$tB_S_f^{(7)} := B_S_f7$$

$$tB_S_f^{(4)} := B_S_f4$$

$$B_S_f := tB_S_f^T$$

$$B_S_f = \begin{pmatrix} -1.95 & 0.356 \\ -1.306 & 0.409 \\ -0.653 & 0.44 \\ 0 & 0.45 \\ 0.653 & 0.44 \\ 1.306 & 0.409 \\ 1.95 & 0.356 \end{pmatrix} \text{cun}$$



V.E.2.b. Ensuring even distances between each Tuning Hole measured along the curvature at the base of the Bridge

Calculation of the Distance from the Axis to Each String Along the Face of the GuQin

$$B_swf_1 := \int_{0cun}^{B_S_f_{1,1}} \sqrt{1 + \left(\frac{d}{dX} B_base(X)\right)^2} dX$$

$$B_swf_2 := \int_{0cun}^{B_S_f_{2,1}} \sqrt{1 + \left(\frac{d}{dX} B_base(X)\right)^2} dX$$

$$B_swf_3 := \int_{0cun}^{B_S_f3,1} \sqrt{1 + \left(\frac{d}{dX}B_base(X)\right)^2} dX$$

$$B_swf_4 := \int_{0cun}^{B_S_f4,1} \sqrt{1 + \left(\frac{d}{dX}B_base(X)\right)^2} dX$$

$$B_swf_5 := \int_{0cun}^{B_S_f5,1} \sqrt{1 + \left(\frac{d}{dX}B_base(X)\right)^2} dX$$

$$B_swf_6 := \int_{0cun}^{B_S_f6,1} \sqrt{1 + \left(\frac{d}{dX}B_base(X)\right)^2} dX$$

$$B_swf_7 := \int_{0cun}^{B_S_f7,1} \sqrt{1 + \left(\frac{d}{dX}B_base(X)\right)^2} dX$$

The distance from the axis to each string

$$B_swf = \begin{pmatrix} -1.952617 \\ -1.3069031 \\ -0.6534555 \\ 0 \\ 0.6534555 \\ 1.3069031 \\ 1.952617 \end{pmatrix} cun$$

VI. Deriving the Curvature Functions for transverse sections of the GuQin

VI.A. Deriving the Curvature Functions from Shoulder to Nut $C_n(x)$ where n =incremental distance from the Shoulder

Inserting uprise toward the tail

$$\begin{pmatrix} 25 & 0 \\ 26 & 0 \\ 27 & 0 \end{pmatrix}$$

$$\text{Uprise} := \begin{pmatrix} 28 & 0 \\ 29 & 0 \\ 29.648 & 0 \\ 39.64797372 & 0.022925775 \\ 49.64788487 & 0.065080186 \\ 59.64771251 & 0.123793003 \\ 69.64744423 & 0.197045233 \\ 79.64707262 & 0.283256765 \\ 89.64659341 & 0.381157299 \end{pmatrix} \cdot \text{cm}$$

$$\text{UpriseCurveConstant} := \text{Ispline}(\text{Uprise}^{\langle 1 \rangle}, \text{Uprise}^{\langle 2 \rangle})$$

$$\text{UC}(X) := \text{interp}(\text{UpriseCurveConstant}, \text{Uprise}^{\langle 1 \rangle}, \text{Uprise}^{\langle 2 \rangle}, X)$$

Derivation of Curvature Functions Cn(X), From Line 07 to Tail, in 1 inch increments

$$\text{C0}(X) := \text{Surface}(X, \text{Width}_7, \text{Height}_c)$$

$$\text{XY1} := \frac{(\text{N}_{\text{XY}} - \text{S}_{\text{XY}})}{\text{Distance}(7, 12)} \cdot 1\text{in} + \text{S}_{\text{XY}} \quad \text{X1} := \text{XY1}^{\langle 1 \rangle} \quad \text{Y1} := \text{XY1}^{\langle 2 \rangle}$$

$$\text{S1} := \text{pspline}(\text{X1}, \text{Y1}) \quad \text{Spline vector constant}$$

$$\text{C1}(X) := \text{interp}(\text{S1}, \text{X1}, \text{Y1}, X) \quad \text{Curvature at 1 inch away from the shoulder, obtained by parabolic interpolation}$$

$$\text{XY2} := \frac{(\text{N}_{\text{XY}} - \text{S}_{\text{XY}})}{\text{Distance}(7, 12)} \cdot 2\text{in} + \text{S}_{\text{XY}} \quad \text{X2} := \text{XY2}^{\langle 1 \rangle} \quad \text{Y2} := \text{XY2}^{\langle 2 \rangle}$$

$$\text{S2} := \text{pspline}(\text{X2}, \text{Y2})$$

$$\text{C2}(X) := \text{interp}(\text{S2}, \text{X2}, \text{Y2}, X)$$

$$\text{XY3} := \frac{(\text{N}_{\text{XY}} - \text{S}_{\text{XY}})}{\text{Distance}(7, 12)} \cdot 3\text{in} + \text{S}_{\text{XY}} \quad \text{X3} := \text{XY3}^{\langle 1 \rangle} \quad \text{Y3} := \text{XY3}^{\langle 2 \rangle}$$

$$\text{S3} := \text{pspline}(\text{X3}, \text{Y3})$$

$$\text{C3}(X) := \text{interp}(\text{S3}, \text{X3}, \text{Y3}, X)$$

$$\text{XY4} := \frac{(\text{N}_{\text{XY}} - \text{S}_{\text{XY}})}{\text{Distance}(7, 12)} \cdot 4\text{in} + \text{S}_{\text{XY}} \quad \text{X4} := \text{XY4}^{\langle 1 \rangle} \quad \text{Y4} := \text{XY4}^{\langle 2 \rangle}$$

$$\text{S4} := \text{pspline}(\text{X4}, \text{Y4})$$

$C4(X) := \text{interp}(S4, X4, Y4, X)$
 $XY5 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 5in + S_XY$ $X5 := XY5^{\langle 1 \rangle}$ $Y5 := XY5^{\langle 2 \rangle}$
 $S5 := \text{pspline}(X5, Y5)$
 $C5(X) := \text{interp}(S5, X5, Y5, X)$
 $XY6 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 6in + S_XY$ $X6 := XY6^{\langle 1 \rangle}$ $Y6 := XY6^{\langle 2 \rangle}$
 $S6 := \text{pspline}(X6, Y6)$
 $C6(X) := \text{interp}(S6, X6, Y6, X)$
 $XY7 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 7in + S_XY$ $X7 := XY7^{\langle 1 \rangle}$ $Y7 := XY7^{\langle 2 \rangle}$
 $S7 := \text{pspline}(X7, Y7)$
 $C7(X) := \text{interp}(S7, X7, Y7, X)$
 $XY8 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 8in + S_XY$ $X8 := XY8^{\langle 1 \rangle}$ $Y8 := XY8^{\langle 2 \rangle}$
 $S8 := \text{pspline}(X8, Y8)$
 $C8(X) := \text{interp}(S8, X8, Y8, X)$
 $XY9 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 9in + S_XY$ $X9 := XY9^{\langle 1 \rangle}$ $Y9 := XY9^{\langle 2 \rangle}$
 $S9 := \text{pspline}(X9, Y9)$
 $C9(X) := \text{interp}(S9, X9, Y9, X)$
 $XY10 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 10in + S_XY$ $X10 := XY10^{\langle 1 \rangle}$ $Y10 := XY10^{\langle 2 \rangle}$
 $S10 := \text{pspline}(X10, Y10)$
 $C10(X) := \text{interp}(S10, X10, Y10, X)$
 $XY11 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 11in + S_XY$ $X11 := XY11^{\langle 1 \rangle}$ $Y11 := XY11^{\langle 2 \rangle}$
 $S11 := \text{pspline}(X11, Y11)$

$$\begin{aligned}
& C11(X) := \text{interp}(S11, X11, Y11, X) \\
XY12 := & \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 12in + S_XY \quad X12 := XY12^{(1)} \quad Y12 := XY12^{(2)} \\
& S12 := \text{pspline}(X12, Y12) \\
& C12(X) := \text{interp}(S12, X12, Y12, X) + UC(12in) \\
XY13 := & \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 13in + S_XY \quad X13 := XY13^{(1)} \quad Y13 := XY13^{(2)} \\
& S13 := \text{pspline}(X13, Y13) \\
& C13(X) := \text{interp}(S13, X13, Y13, X) + UC(13in) \\
XY14 := & \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 14in + S_XY \quad X14 := XY14^{(1)} \quad Y14 := XY14^{(2)} \\
& S14 := \text{pspline}(X14, Y14) \\
& C14(X) := \text{interp}(S14, X14, Y14, X) + UC(14in) \\
XY15 := & \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 15in + S_XY \quad X15 := XY15^{(1)} \quad Y15 := XY15^{(2)} \\
& S15 := \text{pspline}(X15, Y15) \\
& C15(X) := \text{interp}(S15, X15, Y15, X) + UC(15in) \\
XY16 := & \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 16in + S_XY \quad X16 := XY16^{(1)} \quad Y16 := XY16^{(2)} \\
& S16 := \text{pspline}(X16, Y16) \\
& C16(X) := \text{interp}(S16, X16, Y16, X) + UC(16in) \\
XY17 := & \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 17in + S_XY \quad X17 := XY17^{(1)} \quad Y17 := XY17^{(2)} \\
& S17 := \text{pspline}(X17, Y17) \\
& C17(X) := \text{interp}(S17, X17, Y17, X) + UC(17in) \\
XY18 := & \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 18in + S_XY \quad X18 := XY18^{(1)} \quad Y18 := XY18^{(2)} \\
& S18 := \text{pspline}(X18, Y18)
\end{aligned}$$

$$\begin{aligned}
& C18(X) := \text{interp}(S18, X18, Y18, X) + UC(18\text{in}) \\
XY19 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 19\text{in} + S_XY & \quad X19 := XY19^{(1)} \quad Y19 := XY19^{(2)} \\
& S19 := \text{pspline}(X19, Y19) \\
& C19(X) := \text{interp}(S19, X19, Y19, X) + UC(19\text{in}) \\
XY20 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 20\text{in} + S_XY & \quad X20 := XY20^{(1)} \quad Y20 := XY20^{(2)} \\
& S20 := \text{pspline}(X20, Y20) \\
& C20(X) := \text{interp}(S20, X20, Y20, X) + UC(20\text{in}) \\
XY21 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 21\text{in} + S_XY & \quad X21 := XY21^{(1)} \quad Y21 := XY21^{(2)} \\
& S21 := \text{pspline}(X21, Y21) \\
& C21(X) := \text{interp}(S21, X21, Y21, X) + UC(21\text{in}) \\
XY22 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 22\text{in} + S_XY & \quad X22 := XY22^{(1)} \quad Y22 := XY22^{(2)} \\
& S22 := \text{pspline}(X22, Y22) \\
& C22(X) := \text{interp}(S22, X22, Y22, X) + UC(22\text{in}) \\
XY23 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 23\text{in} + S_XY & \quad X23 := XY23^{(1)} \quad Y23 := XY23^{(2)} \\
& S23 := \text{pspline}(X23, Y23) \\
& C23(X) := \text{interp}(S23, X23, Y23, X) + UC(23\text{in}) \\
XY24 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 24\text{in} + S_XY & \quad X24 := XY24^{(1)} \quad Y24 := XY24^{(2)} \\
& S24 := \text{pspline}(X24, Y24) \\
& C24(X) := \text{interp}(S24, X24, Y24, X) + UC(24\text{in}) \\
XY25 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 25\text{in} + S_XY & \quad X25 := XY25^{(1)} \quad Y25 := XY25^{(2)} \\
& S25 := \text{pspline}(X25, Y25) \\
& C25(X) := \text{interp}(S25, X25, Y25, X) + UC(25\text{in})
\end{aligned}$$

$$XY26 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 26\text{in} + S_XY \quad X26 := XY26^{\langle 1 \rangle} \quad Y26 := XY26^{\langle 2 \rangle}$$

$$S26 := \text{pspline}(X26, Y26)$$

$$C26(X) := \text{interp}(S26, X26, Y26, X) + \text{UC}(26\text{in})$$

$$XY27 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 27\text{in} + S_XY \quad X27 := XY27^{\langle 1 \rangle} \quad Y27 := XY27^{\langle 2 \rangle}$$

$$S27 := \text{pspline}(X27, Y27)$$

$$C27(X) := \text{interp}(S27, X27, Y27, X) + \text{UC}(27\text{in})$$

$$XY28 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 28\text{in} + S_XY \quad X28 := XY28^{\langle 1 \rangle} \quad Y28 := XY28^{\langle 2 \rangle}$$

$$S28 := \text{pspline}(X28, Y28)$$

$$C28(X) := \text{interp}(S28, X28, Y28, X) + \text{UC}(28\text{in})$$

$$XY29 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 29\text{in} + S_XY \quad X29 := XY29^{\langle 1 \rangle} \quad Y29 := XY29^{\langle 2 \rangle}$$

$$S29 := \text{pspline}(X29, Y29)$$

$$C29(X) := \text{interp}(S29, X29, Y29, X) + \text{UC}(29\text{in})$$

$$XY30 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 30\text{in} + S_XY \quad X30 := XY30^{\langle 1 \rangle} \quad Y30 := XY30^{\langle 2 \rangle}$$

$$S30 := \text{pspline}(X30, Y30)$$

$$C30(X) := \text{interp}(S30, X30, Y30, X) + \text{UC}(30\text{in})$$

$$XY31 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 31\text{in} + S_XY \quad X31 := XY31^{\langle 1 \rangle} \quad Y31 := XY31^{\langle 2 \rangle}$$

$$S31 := \text{pspline}(X31, Y31)$$

$$C31(X) := \text{interp}(S31, X31, Y31, X) + \text{UC}(31\text{in})$$

$$XY32 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 32\text{in} + S_XY \quad X32 := XY32^{\langle 1 \rangle} \quad Y32 := XY32^{\langle 2 \rangle}$$

$$S32 := \text{pspline}(X32, Y32)$$

$$C32(X) := \text{interp}(S32, X32, Y32, X) + \text{UC}(32\text{in})$$

$$XY33 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 33\text{in} + S_XY \quad X33 := XY33^{(1)} \quad Y33 := XY33^{(2)}$$

$$S33 := \text{pspline}(X33, Y33)$$

$$C33(X) := \text{interp}(S33, X33, Y33, X) + UC(33\text{in})$$

$$XY34 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 34\text{in} + S_XY \quad X34 := XY34^{(1)} \quad Y34 := XY34^{(2)}$$

$$S34 := \text{pspline}(X34, Y34)$$

$$C34(X) := \text{interp}(S34, X34, Y34, X) + UC(34\text{in})$$

$$XY35 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot 35\text{in} + S_XY \quad X35 := XY35^{(1)} \quad Y35 := XY35^{(2)}$$

$$S35 := \text{pspline}(X35, Y35)$$

$$C35(X) := \text{interp}(S35, X35, Y35, X) + UC(35\text{in})$$

ORIGIN := 0

$$p := \begin{pmatrix} -4 \\ -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

TD0 := C0(p·in)	TD1 := C1(p in)	TD2 := C2(p in)	TD3 := C3(p in)
TD4 := C4(p in)	TD5 := C5(p in)	TD6 := C6(p in)	TD7 := C7(p in)
TD8 := C8(p in)	TD9 := C9(p in)	TD10 := C10(p in)	TD11 := C11(p in)
TD12 := C12(p in)	TD13 := C13(p in)	TD14 := C14(p in)	TD15 := C15(p in)
TD16 := C16(p in)	TD17 := C17(p in)	TD18 := C18(p in)	TD19 := C19(p in)
TD20 := C20(p in)	TD21 := C21(p in)	TD22 := C22(p in)	TD23 := C23(p in)
TD24 := C24(p in)	TD25 := C25(p in)	TD26 := C26(p in)	TD27 := C27(p in)
TD28 := C28(p·in)	TD29 := C29(p in)	TD30 := C30(p in)	TD31 := C31(p in)

TD32 := C32(p in) TD33 := C33(p in) TD34 := C34(p in) TD35 := C35(p in)

TD^{<1>} := TD1

TD^{<2>} := TD2

TD^{<3>} := TD3

TD^{<4>} := TD4

TD^{<5>} := TD5

TD^{<6>} := TD6

TD^{<7>} := TD7

TD^{<8>} := TD8

TD^{<9>} := TD9

TD^{<10>} := TD10

TD^{<11>} := TD11

TD^{<12>} := TD12

TD^{<13>} := TD13

TD^{<14>} := TD14

TD^{<15>} := TD15

TD^{<16>} := TD16

TD^{<17>} := TD17

TD^{<18>} := TD18

TD^{<19>} := TD19

TD^{<20>} := TD20

TD^{<21>} := TD21

TD^{<22>} := TD22

TD^{<23>} := TD23

TD^{<24>} := TD24

TD^{<25>} := TD25

TD^{<26>} := TD26

TD^{<27>} := TD27

TD^{<28>} := TD28

TD^{<29>} := TD29

TD^{<30>} := TD30

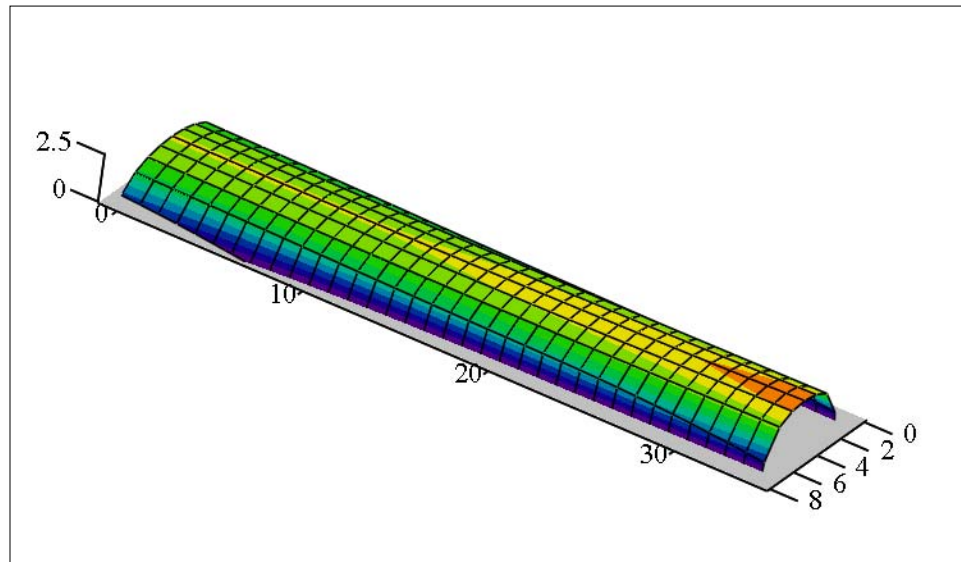
TD^{<31>} := TD31

TD^{<32>} := TD32

TD^{<33>} := TD33

TD^{<34>} := TD34

TD^{<35>} := TD35



TD

VI.B. Deriving the Curvature Functions from Shoulder to the First Hui
 $rC_n(x)$ where n =incremental distance from the Shoulder

ORIGIN := 1

Derivation of Curvature Functions Crn(X), From Line 07 to First Hui, in 1 inch increments

$$D_BtoHui1 := \frac{\text{Distance}(4, 12)}{8}$$

$$D_toHui1 := \text{Dist}_4 + D_BtoHui1$$

$$D_toHui1 = 7.2 \text{ cun}$$

$$\text{Dist}_7 - D_toHui1 = 3.3 \text{ in}$$

$$rXY1 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot -1 \text{ in} + S_XY \quad rX1 := rXY1^{(1)} \quad rY1 := rXY1^{(2)}$$

$$rS1 := \text{pspline}(rX1, rY1)$$

$$rC1(X) := \text{interp}(rS1, rX1, rY1, X)$$

$$rXY2 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot -2 \text{ in} + S_XY \quad rX2 := rXY2^{(1)} \quad rY2 := rXY2^{(2)}$$

$$rS2 := \text{pspline}(rX2, rY2)$$

$$rC2(X) := \text{interp}(rS2, rX2, rY2, X)$$

$$rXY3 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot -3 \text{ in} + S_XY \quad rX3 := rXY3^{(1)} \quad rY3 := rXY3^{(2)}$$

$$rS3 := \text{pspline}(rX3, rY3)$$

$$rC3(X) := \text{interp}(rS3, rX3, rY3, X)$$

VI.C. Deriving the Curvature Function at the First Hui
 $CHui1(x)$

$$XYHui1 := \frac{(N_XY - S_XY)}{\text{Distance}(7, 12)} \cdot -3.3 \text{ in} + S_XY \quad XHui1 := XYHui1^{(1)} \quad YHui1 := XYHui1^{(2)}$$

$$SHui1 := \text{pspline}(XHui1, YHui1)$$

$$CHui1(X) := \text{interp}(SHui1, XHui1, YHui1, X)$$

VI.D. Deriving the Curvature Functions from Bridge to the First Hui
CBH $n(x)$ where n =incremental distance from the Bridge

Derivation of Curvature Functions CBH $n(X)$, From Bridge to First Hui, in 1 inch increments

$$\text{interp_XYBH1} := \frac{(S_XY - N_XY)}{\text{Distance}(7, 12)} \cdot (\text{Distance}(4, 12) - 1\text{in}) + N_XY$$

$$\text{XBH1} := \text{interp_XYBH1}^{\langle 1 \rangle} \quad \text{YBH1} := \text{interp_XYBH1}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(\text{D_BtoHui1} - 1\text{in})}{\text{D_BtoHui1}}$$

$$\text{SBH1} := \text{pspline}(\text{XBH1}, \text{YBH1})$$

$$\text{CBH1}(X) := \text{interp}(\text{SBH1}, \text{XBH1}, \text{YBH1}, X)$$

$$\text{interp_XYBH2} := \frac{(S_XY - N_XY)}{\text{Distance}(7, 12)} \cdot (\text{Distance}(4, 12) - 2\text{in}) + N_XY$$

$$\text{XBH2} := \text{interp_XYBH2}^{\langle 1 \rangle} \quad \text{YBH2} := \text{interp_XYBH2}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(\text{D_BtoHui1} - 2\text{in})}{\text{D_BtoHui1}}$$

$$\text{SBH2} := \text{pspline}(\text{XBH2}, \text{YBH2})$$

$$\text{CBH2}(X) := \text{interp}(\text{SBH2}, \text{XBH2}, \text{YBH2}, X)$$

$$\text{interp_XYBH3} := \frac{(S_XY - N_XY)}{\text{Distance}(7, 12)} \cdot (\text{Distance}(4, 12) - 3\text{in}) + N_XY$$

$$\text{XBH3} := \text{interp_XYBH3}^{\langle 1 \rangle} \quad \text{YBH3} := \text{interp_XYBH3}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(\text{D_BtoHui1} - 3\text{in})}{\text{D_BtoHui1}}$$

$$\text{SBH3} := \text{pspline}(\text{XBH3}, \text{YBH3})$$

$$\text{CBH3}(X) := \text{interp}(\text{SBH3}, \text{XBH3}, \text{YBH3}, X)$$

$$\text{interp_XYBH4} := \frac{(S_XY - N_XY)}{\text{Distance}(7, 12)} \cdot (\text{Distance}(4, 12) - 4\text{in}) + N_XY$$

$$\text{XBH4} := \text{interp_XYBH4}^{\langle 1 \rangle} \quad \text{YBH4} := \text{interp_XYBH4}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(\text{D_BtoHui1} - 4\text{in})}{\text{D_BtoHui1}}$$

$$\text{SBH4} := \text{pspline}(\text{XBH4}, \text{YBH4})$$

$$\text{CBH4}(X) := \text{interp}(\text{SBH4}, \text{XBH4}, \text{YBH4}, X)$$

$$\text{interp_XYBH5} := \frac{(S_XY - N_XY)}{\text{Distance}(7, 12)} \cdot (\text{Distance}(4, 12) - 5\text{in}) + N_XY$$

$$\text{XBH5} := \text{interp_XYBH5}^{\langle 1 \rangle} \quad \text{YBH5} := \text{interp_XYBH5}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(\text{D_BtoHui1} - 5\text{in})}{\text{D_BtoHui1}}$$

$$\text{SBH5} := \text{pspline}(\text{XBH5}, \text{YBH5})$$

$$\text{CBH5}(X) := \text{interp}(\text{SBH5}, \text{XBH5}, \text{YBH5}, X)$$

VI.E. Including the Curvature Function at the base of the Bridge
B_base(x)

(See section V.D.)

VII. Calculating drilling depths of $1/16$ inch increments

Solving for Drilling Depth Locations

$$d := \frac{1}{16} \text{in} \quad \text{Wood_width} := 0.65 \text{cun} \quad \text{inc} := 1 \dots 10$$

$$d_{\text{inc}} := d \cdot \text{inc} \quad x_{\text{inc}} := 1.5 \text{cun}$$

VII.A. Specifying positions of each drilling depth from Shoulder to Nut
Depth_Shoulder_to_Nut

From Shoulder to Nut, in 1 inch intervals

$$\text{Drill} := \frac{1}{32} \text{in} \quad \text{WTh} := 2.4 \text{cm} \quad \text{incr} := 1 \dots 36$$

$$\text{Dr}_{\text{incr}} := \text{Drill} \cdot \text{incr} \quad \text{ID} := 2 \cdot \text{cm}$$

$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_1 - \text{C35}(\text{ID}), \text{ID})$	ID = 2.013 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_2 - \text{C35}(\text{ID}), \text{ID})$	ID = 2.902 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_3 - \text{C35}(\text{ID}), \text{ID})$	ID = 3.554 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_4 - \text{C35}(\text{ID}), \text{ID})$	ID = 4.073 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_5 - \text{C35}(\text{ID}), \text{ID})$	ID = 4.52 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_6 - \text{C35}(\text{ID}), \text{ID})$	ID = 4.913 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_7 - \text{C35}(\text{ID}), \text{ID})$	ID = 5.255 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_8 - \text{C35}(\text{ID}), \text{ID})$	ID = 5.547 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_9 - \text{C35}(\text{ID}), \text{ID})$	ID = 5.808 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{10} - \text{C35}(\text{ID}), \text{ID})$	ID = 6.061 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{11} - \text{C35}(\text{ID}), \text{ID})$	ID = 6.313 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{12} - \text{C35}(\text{ID}), \text{ID})$	ID = 6.541 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{13} - \text{C35}(\text{ID}), \text{ID})$	ID = 6.717 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{14} - \text{C35}(\text{ID}), \text{ID})$	ID = 6.854 cm

$ID := \text{root}(WTh - Dr_{15} - C35(ID), ID)$	ID = 6.965 cm
$ID := \text{root}(WTh - Dr_{16} - C35(ID), ID)$	ID = 7.061 cm
$ID := \text{root}(WTh - Dr_{17} - C35(ID), ID)$	ID = 7.146 cm
$ID := \text{root}(WTh - Dr_{18} - C35(ID), ID)$	ID = 7.223 cm
$ID := \text{root}(WTh - Dr_{19} - C35(ID), ID)$	ID = 7.294 cm
$ID := \text{root}(WTh - Dr_{20} - C35(ID), ID)$	ID = 7.36 cm
$ID := \text{root}(WTh - Dr_{21} - C35(ID), ID)$	ID = 7.422 cm
$ID := \text{root}(WTh - Dr_{22} - C35(ID), ID)$	ID = 7.481 cm
$ID := \text{root}(WTh - Dr_{24} - C35(ID), ID)$	ID = 7.59 cm
$ID := \text{root}(WTh - Dr_{26} - C35(ID), ID)$	ID = 7.691 cm
$ID := \text{root}(WTh - Dr_{30} - C35(ID), ID)$	ID = 7.873 cm
$ID := \text{root}(WTh - Dr_{34} - C35(ID), ID)$	ID = 8.036 cm

ID := 1.624cm

$ID := \text{root}(WTh - Dr_1 - C34(ID), ID)$	ID = 1.624 cm
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ID := 1.12cm

$ID := \text{root}(WTh - Dr_1 - C33(ID), ID)$	ID = 1.106 cm
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ID := 2.1cm

$ID := \text{root}(WTh - Dr_2 - C32(ID), ID)$	ID = 2.117 cm
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ID := 1.3cm

$ID := \text{root}(WTh - Dr_2 - C30(ID), ID)$	ID = 1.349 cm
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$ID := \text{root}(WTh - Dr_3 - C30(ID), ID)$	ID = 2.598 cm
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$ID := \text{root}(WTh - Dr_4 - C30(ID), ID)$	ID = 3.432 cm
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$ID := \text{root}(WTh - Dr_5 - C30(ID), ID)$	ID = 4.061 cm
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$ID := \text{root}(WTh - Dr_6 - C30(ID), ID)$	ID = 4.572 cm
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$ID := \text{root}(WTh - Dr_7 - C30(ID), ID)$	ID = 5.014 cm
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$ID := \text{root}(WTh - Dr_8 - C30(ID), ID)$	ID = 5.405 cm
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$ID := \text{root}(WTh - Dr_9 - C30(ID), ID)$	ID = 5.746 cm
$ID := \text{root}(WTh - Dr_{10} - C30(ID), ID)$	ID = 6.038 cm
$ID := \text{root}(WTh - Dr_{11} - C30(ID), ID)$	ID = 6.3 cm
$ID := \text{root}(WTh - Dr_{12} - C30(ID), ID)$	ID = 6.554 cm
$ID := \text{root}(WTh - Dr_{13} - C30(ID), ID)$	ID = 6.807 cm
$ID := \text{root}(WTh - Dr_{14} - C30(ID), ID)$	ID = 7.033 cm
$ID := \text{root}(WTh - Dr_{15} - C30(ID), ID)$	ID = 7.205 cm
$ID := \text{root}(WTh - Dr_{16} - C30(ID), ID)$	ID = 7.34 cm
$ID := \text{root}(WTh - Dr_{17} - C30(ID), ID)$	ID = 7.449 cm
$ID := \text{root}(WTh - Dr_{18} - C30(ID), ID)$	ID = 7.543 cm
$ID := \text{root}(WTh - Dr_{19} - C30(ID), ID)$	ID = 7.627 cm
$ID := \text{root}(WTh - Dr_{20} - C30(ID), ID)$	ID = 7.704 cm
$ID := \text{root}(WTh - Dr_{21} - C30(ID), ID)$	ID = 7.774 cm
$ID := \text{root}(WTh - Dr_{22} - C30(ID), ID)$	ID = 7.84 cm
$ID := \text{root}(WTh - Dr_{24} - C30(ID), ID)$	ID = 7.96 cm
$ID := \text{root}(WTh - Dr_{26} - C30(ID), ID)$	ID = 8.07 cm
$ID := \text{root}(WTh - Dr_{30} - C30(ID), ID)$	ID = 8.265 cm
$ID := \text{root}(WTh - Dr_{34} - C30(ID), ID)$	ID = 8.436 cm

ID := 0.7cm

$ID := \text{root}(WTh - Dr_2 - C29(ID), ID)$	ID = 0.66 cm
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ID := 1.7cm

$ID := \text{root}(WTh - Dr_3 - C27(ID), ID)$	ID = 1.73 cm
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ID := 1.3cm

$ID := \text{root}(WTh - Dr_3 - C26(ID), ID)$	ID = 1.327 cm
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ID := 0.7cm

$ID := \text{root}(WTh - Dr_3 - C25(ID), ID)$	ID = 0.677 cm
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$ID := \text{root}(WTh - Dr_4 - C25(ID), ID)$	ID = 2.418 cm
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$ID := \text{root}(WTh - Dr_5 - C25(ID), ID)$	ID = 3.399 cm
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$ID := \text{root}(WTh - Dr_6 - C25(ID), ID)$	ID = 4.125 cm
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$ID := \text{root}(WTh - Dr_7 - C25(ID), ID)$	ID = 4.699 cm
$ID := \text{root}(WTh - Dr_8 - C25(ID), ID)$	ID = 5.186 cm
$ID := \text{root}(WTh - Dr_9 - C25(ID), ID)$	ID = 5.611 cm
$ID := \text{root}(WTh - Dr_{10} - C25(ID), ID)$	ID = 5.988 cm
$ID := \text{root}(WTh - Dr_{11} - C25(ID), ID)$	ID = 6.315 cm
$ID := \text{root}(WTh - Dr_{12} - C25(ID), ID)$	ID = 6.597 cm
$ID := \text{root}(WTh - Dr_{13} - C25(ID), ID)$	ID = 6.856 cm
$ID := \text{root}(WTh - Dr_{14} - C25(ID), ID)$	ID = 7.111 cm
$ID := \text{root}(WTh - Dr_{15} - C25(ID), ID)$	ID = 7.366 cm
$ID := \text{root}(WTh - Dr_{16} - C25(ID), ID)$	ID = 7.572 cm
$ID := \text{root}(WTh - Dr_{17} - C25(ID), ID)$	ID = 7.729 cm
$ID := \text{root}(WTh - Dr_{18} - C25(ID), ID)$	ID = 7.853 cm
$ID := \text{root}(WTh - Dr_{19} - C25(ID), ID)$	ID = 7.957 cm
$ID := \text{root}(WTh - Dr_{20} - C25(ID), ID)$	ID = 8.048 cm
$ID := \text{root}(WTh - Dr_{21} - C25(ID), ID)$	ID = 8.129 cm
$ID := \text{root}(WTh - Dr_{22} - C25(ID), ID)$	ID = 8.203 cm
$ID := \text{root}(WTh - Dr_{23} - C25(ID), ID)$	ID = 8.272 cm
$ID := \text{root}(WTh - Dr_{24} - C25(ID), ID)$	ID = 8.337 cm
$ID := \text{root}(WTh - Dr_{25} - C25(ID), ID)$	ID = 8.398 cm
$ID := \text{root}(WTh - Dr_{26} - C25(ID), ID)$	ID = 8.456 cm
$ID := \text{root}(WTh - Dr_{30} - C25(ID), ID)$	ID = 8.664 cm
$ID := \text{root}(WTh - Dr_{34} - C25(ID), ID)$	ID = 8.845 cm

ID := 1.25cm

$ID := \text{root}(WTh - Dr_4 - C21(ID), ID)$	ID = 1.259 cm
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ID := 0.7cm

$ID := \text{root}(WTh - Dr_4 - C20(ID), ID)$	ID = 0.709 cm
$ID := \text{root}(WTh - Dr_5 - C20(ID), ID)$	ID = 2.561 cm
$ID := \text{root}(WTh - Dr_6 - C20(ID), ID)$	ID = 3.556 cm
$ID := \text{root}(WTh - Dr_7 - C20(ID), ID)$	ID = 4.34 cm
$ID := \text{root}(WTh - Dr_8 - C20(ID), ID)$	ID = 4.949 cm
$ID := \text{root}(WTh - Dr_9 - C20(ID), ID)$	ID = 5.467 cm
$ID := \text{root}(WTh - Dr_{10} - C20(ID), ID)$	ID = 5.913 cm
$ID := \text{root}(WTh - Dr_{11} - C20(ID), ID)$	ID = 6.31 cm

$ID := \text{root}(WTh - Dr_{12} - C20(ID), ID)$	ID = 6.659 cm
$ID := \text{root}(WTh - Dr_{13} - C20(ID), ID)$	ID = 6.96 cm
$ID := \text{root}(WTh - Dr_{14} - C20(ID), ID)$	ID = 7.228 cm
$ID := \text{root}(WTh - Dr_{15} - C20(ID), ID)$	ID = 7.487 cm
$ID := \text{root}(WTh - Dr_{16} - C20(ID), ID)$	ID = 7.744 cm
$ID := \text{root}(WTh - Dr_{17} - C20(ID), ID)$	ID = 7.975 cm
$ID := \text{root}(WTh - Dr_{18} - C20(ID), ID)$	ID = 8.149 cm
$ID := \text{root}(WTh - Dr_{19} - C20(ID), ID)$	ID = 8.284 cm
$ID := \text{root}(WTh - Dr_{20} - C20(ID), ID)$	ID = 8.394 cm
$ID := \text{root}(WTh - Dr_{21} - C20(ID), ID)$	ID = 8.489 cm
$ID := \text{root}(WTh - Dr_{22} - C20(ID), ID)$	ID = 8.574 cm
$ID := \text{root}(WTh - Dr_{23} - C20(ID), ID)$	ID = 8.651 cm
$ID := \text{root}(WTh - Dr_{24} - C20(ID), ID)$	ID = 8.722 cm
$ID := \text{root}(WTh - Dr_{25} - C20(ID), ID)$	ID = 8.788 cm
$ID := \text{root}(WTh - Dr_{26} - C20(ID), ID)$	ID = 8.851 cm
$ID := \text{root}(WTh - Dr_{27} - C20(ID), ID)$	ID = 8.91 cm
$ID := \text{root}(WTh - Dr_{28} - C20(ID), ID)$	ID = 8.966 cm
$ID := \text{root}(WTh - Dr_{29} - C20(ID), ID)$	ID = 9.02 cm
$ID := \text{root}(WTh - Dr_{30} - C20(ID), ID)$	ID = 9.072 cm
$ID := \text{root}(WTh - Dr_{34} - C20(ID), ID)$	ID = 9.262 cm

ID := 1.69cm

$ID := \text{root}(WTh - Dr_5 - C15(ID), ID)$	ID = 1.694 cm
$ID := \text{root}(WTh - Dr_6 - C15(ID), ID)$	ID = 3.079 cm
$ID := \text{root}(WTh - Dr_7 - C15(ID), ID)$	ID = 4.004 cm
$ID := \text{root}(WTh - Dr_8 - C15(ID), ID)$	ID = 4.775 cm
$ID := \text{root}(WTh - Dr_9 - C15(ID), ID)$	ID = 5.38 cm
$ID := \text{root}(WTh - Dr_{10} - C15(ID), ID)$	ID = 5.899 cm
$ID := \text{root}(WTh - Dr_{11} - C15(ID), ID)$	ID = 6.348 cm
$ID := \text{root}(WTh - Dr_{12} - C15(ID), ID)$	ID = 6.749 cm
$ID := \text{root}(WTh - Dr_{13} - C15(ID), ID)$	ID = 7.104 cm
$ID := \text{root}(WTh - Dr_{14} - C15(ID), ID)$	ID = 7.411 cm
$ID := \text{root}(WTh - Dr_{15} - C15(ID), ID)$	ID = 7.683 cm

$ID := \text{root}(WTh - Dr_{16} - C15(ID), ID)$	ID = 7.945 cm
$ID := \text{root}(WTh - Dr_{17} - C15(ID), ID)$	ID = 8.207 cm
$ID := \text{root}(WTh - Dr_{18} - C15(ID), ID)$	ID = 8.44 cm
$ID := \text{root}(WTh - Dr_{19} - C15(ID), ID)$	ID = 8.616 cm
$ID := \text{root}(WTh - Dr_{20} - C15(ID), ID)$	ID = 8.753 cm
$ID := \text{root}(WTh - Dr_{21} - C15(ID), ID)$	ID = 8.863 cm
$ID := \text{root}(WTh - Dr_{22} - C15(ID), ID)$	ID = 8.959 cm
$ID := \text{root}(WTh - Dr_{23} - C15(ID), ID)$	ID = 9.045 cm
$ID := \text{root}(WTh - Dr_{24} - C15(ID), ID)$	ID = 9.122 cm
$ID := \text{root}(WTh - Dr_{25} - C15(ID), ID)$	ID = 9.194 cm
$ID := \text{root}(WTh - Dr_{26} - C15(ID), ID)$	ID = 9.261 cm
$ID := \text{root}(WTh - Dr_{27} - C15(ID), ID)$	ID = 9.323 cm
$ID := \text{root}(WTh - Dr_{28} - C15(ID), ID)$	ID = 9.383 cm
$ID := \text{root}(WTh - Dr_{29} - C15(ID), ID)$	ID = 9.44 cm
$ID := \text{root}(WTh - Dr_{30} - C15(ID), ID)$	ID = 9.494 cm
$ID := \text{root}(WTh - Dr_{31} - C15(ID), ID)$	ID = 9.546 cm
$ID := \text{root}(WTh - Dr_{32} - C15(ID), ID)$	ID = 9.597 cm
$ID := \text{root}(WTh - Dr_{33} - C15(ID), ID)$	ID = 9.645 cm
$ID := \text{root}(WTh - Dr_{34} - C15(ID), ID)$	ID = 9.692 cm
$ID := \text{root}(WTh - Dr_{35} - C15(ID), ID)$	ID = 9.738 cm

ID := 1.2cm

$ID := \text{root}(WTh - Dr_5 - C10(ID), ID)$	ID = 1.2 cm
$ID := \text{root}(WTh - Dr_6 - C10(ID), ID)$	ID = 3.009 cm
$ID := \text{root}(WTh - Dr_7 - C10(ID), ID)$	ID = 4.01 cm
$ID := \text{root}(WTh - Dr_8 - C10(ID), ID)$	ID = 4.823 cm
$ID := \text{root}(WTh - Dr_9 - C10(ID), ID)$	ID = 5.5 cm
$ID := \text{root}(WTh - Dr_{10} - C10(ID), ID)$	ID = 6.058 cm
$ID := \text{root}(WTh - Dr_{11} - C10(ID), ID)$	ID = 6.544 cm
$ID := \text{root}(WTh - Dr_{12} - C10(ID), ID)$	ID = 6.971 cm
$ID := \text{root}(WTh - Dr_{13} - C10(ID), ID)$	ID = 7.357 cm
$ID := \text{root}(WTh - Dr_{14} - C10(ID), ID)$	ID = 7.696 cm
$ID := \text{root}(WTh - Dr_{15} - C10(ID), ID)$	ID = 7.989 cm
$ID := \text{root}(WTh - Dr_{16} - C10(ID), ID)$	ID = 8.257 cm

$ID := \text{root}(WTh - Dr_{17} - C10(ID), ID)$	ID = 8.519 cm
$ID := \text{root}(WTh - Dr_{18} - C10(ID), ID)$	ID = 8.781 cm
$ID := \text{root}(WTh - Dr_{19} - C10(ID), ID)$	ID = 8.992 cm
$ID := \text{root}(WTh - Dr_{20} - C10(ID), ID)$	ID = 9.15 cm
$ID := \text{root}(WTh - Dr_{21} - C10(ID), ID)$	ID = 9.274 cm
$ID := \text{root}(WTh - Dr_{22} - C10(ID), ID)$	ID = 9.379 cm
$ID := \text{root}(WTh - Dr_{23} - C10(ID), ID)$	ID = 9.47 cm
$ID := \text{root}(WTh - Dr_{24} - C10(ID), ID)$	ID = 9.552 cm
$ID := \text{root}(WTh - Dr_{25} - C10(ID), ID)$	ID = 9.627 cm
$ID := \text{root}(WTh - Dr_{26} - C10(ID), ID)$	ID = 9.697 cm
$ID := \text{root}(WTh - Dr_{27} - C10(ID), ID)$	ID = 9.762 cm
$ID := \text{root}(WTh - Dr_{28} - C10(ID), ID)$	ID = 9.824 cm
$ID := \text{root}(WTh - Dr_{29} - C10(ID), ID)$	ID = 9.883 cm
$ID := \text{root}(WTh - Dr_{30} - C10(ID), ID)$	ID = 9.939 cm
$ID := \text{root}(WTh - Dr_{31} - C10(ID), ID)$	ID = 9.993 cm
$ID := \text{root}(WTh - Dr_{32} - C10(ID), ID)$	ID = 10.044 cm
$ID := \text{root}(WTh - Dr_{33} - C10(ID), ID)$	ID = 10.094 cm
$ID := \text{root}(WTh - Dr_{34} - C10(ID), ID)$	ID = 10.142 cm
$ID := \text{root}(WTh - Dr_{35} - C10(ID), ID)$	ID = 10.189 cm

ID := 1.258cm

$ID := \text{root}(WTh - Dr_5 - C5(ID), ID)$	ID = 1.257 cm
$ID := \text{root}(WTh - Dr_6 - C5(ID), ID)$	ID = 3.156 cm
$ID := \text{root}(WTh - Dr_7 - C5(ID), ID)$	ID = 4.209 cm
$ID := \text{root}(WTh - Dr_8 - C5(ID), ID)$	ID = 5.035 cm
$ID := \text{root}(WTh - Dr_9 - C5(ID), ID)$	ID = 5.73 cm
$ID := \text{root}(WTh - Dr_{10} - C5(ID), ID)$	ID = 6.318 cm
$ID := \text{root}(WTh - Dr_{11} - C5(ID), ID)$	ID = 6.826 cm
$ID := \text{root}(WTh - Dr_{12} - C5(ID), ID)$	ID = 7.273 cm
$ID := \text{root}(WTh - Dr_{13} - C5(ID), ID)$	ID = 7.675 cm
$ID := \text{root}(WTh - Dr_{14} - C5(ID), ID)$	ID = 8.035 cm
$ID := \text{root}(WTh - Dr_{15} - C5(ID), ID)$	ID = 8.348 cm
$ID := \text{root}(WTh - Dr_{16} - C5(ID), ID)$	ID = 8.626 cm
$ID := \text{root}(WTh - Dr_{17} - C5(ID), ID)$	ID = 8.893 cm

$ID := \text{root}(WTh - Dr_{18} - C5(ID), ID)$	ID = 9.158 cm
$ID := \text{root}(WTh - Dr_{19} - C5(ID), ID)$	ID = 9.395 cm
$ID := \text{root}(WTh - Dr_{20} - C5(ID), ID)$	ID = 9.57 cm
$ID := \text{root}(WTh - Dr_{21} - C5(ID), ID)$	ID = 9.706 cm
$ID := \text{root}(WTh - Dr_{22} - C5(ID), ID)$	ID = 9.816 cm
$ID := \text{root}(WTh - Dr_{23} - C5(ID), ID)$	ID = 9.912 cm
$ID := \text{root}(WTh - Dr_{24} - C5(ID), ID)$	ID = 9.998 cm
$ID := \text{root}(WTh - Dr_{25} - C5(ID), ID)$	ID = 10.075 cm
$ID := \text{root}(WTh - Dr_{26} - C5(ID), ID)$	ID = 10.147 cm
$ID := \text{root}(WTh - Dr_{27} - C5(ID), ID)$	ID = 10.215 cm
$ID := \text{root}(WTh - Dr_{28} - C5(ID), ID)$	ID = 10.278 cm
$ID := \text{root}(WTh - Dr_{29} - C5(ID), ID)$	ID = 10.338 cm
$ID := \text{root}(WTh - Dr_{30} - C5(ID), ID)$	ID = 10.396 cm
$ID := \text{root}(WTh - Dr_{31} - C5(ID), ID)$	ID = 10.45 cm
$ID := \text{root}(WTh - Dr_{32} - C5(ID), ID)$	ID = 10.503 cm
$ID := \text{root}(WTh - Dr_{33} - C5(ID), ID)$	ID = 10.554 cm
$ID := \text{root}(WTh - Dr_{34} - C5(ID), ID)$	ID = 10.603 cm
$ID := \text{root}(WTh - Dr_{35} - C5(ID), ID)$	ID = 10.651 cm

ID := 1.313cm

$ID := \text{root}(WTh - Dr_5 - C0(ID), ID)$	ID = 1.312 cm
$ID := \text{root}(WTh - Dr_6 - C0(ID), ID)$	ID = 3.292 cm
$ID := \text{root}(WTh - Dr_7 - C0(ID), ID)$	ID = 4.407 cm
$ID := \text{root}(WTh - Dr_8 - C0(ID), ID)$	ID = 5.266 cm
$ID := \text{root}(WTh - Dr_9 - C0(ID), ID)$	ID = 5.968 cm
$ID := \text{root}(WTh - Dr_{10} - C0(ID), ID)$	ID = 6.573 cm
$ID := \text{root}(WTh - Dr_{11} - C0(ID), ID)$	ID = 7.102 cm
$ID := \text{root}(WTh - Dr_{12} - C0(ID), ID)$	ID = 7.569 cm
$ID := \text{root}(WTh - Dr_{13} - C0(ID), ID)$	ID = 7.986 cm
$ID := \text{root}(WTh - Dr_{14} - C0(ID), ID)$	ID = 8.36 cm
$ID := \text{root}(WTh - Dr_{15} - C0(ID), ID)$	ID = 8.697 cm
$ID := \text{root}(WTh - Dr_{16} - C0(ID), ID)$	ID = 9.001 cm
$ID := \text{root}(WTh - Dr_{17} - C0(ID), ID)$	ID = 9.276 cm
$ID := \text{root}(WTh - Dr_{18} - C0(ID), ID)$	ID = 9.523 cm
$ID := \text{root}(WTh - Dr_{19} - C0(ID), ID)$	ID = 9.746 cm

$ID := \text{root}(WTh - Dr_{20} - C0(ID), ID)$	ID = 9.945 cm
$ID := \text{root}(WTh - Dr_{21} - C0(ID), ID)$	ID = 10.122 cm
$ID := \text{root}(WTh - Dr_{22} - C0(ID), ID)$	ID = 10.279 cm
$ID := \text{root}(WTh - Dr_{23} - C0(ID), ID)$	ID = 10.415 cm
$ID := \text{root}(WTh - Dr_{24} - C0(ID), ID)$	ID = 10.533 cm
$ID := \text{root}(WTh - Dr_{25} - C0(ID), ID)$	ID = 10.632 cm
$ID := \text{root}(WTh - Dr_{26} - C0(ID), ID)$	ID = 10.713 cm
$ID := \text{root}(WTh - Dr_{27} - C0(ID), ID)$	ID = 10.777 cm
$ID := \text{root}(WTh - Dr_{28} - C0(ID), ID)$	ID = 10.823 cm
$ID := \text{root}(WTh - Dr_{29} - C0(ID), ID)$	ID = 10.852 cm
$ID := \text{root}(WTh - Dr_{30} - C0(ID), ID)$	ID = 10.865 cm

$$\text{AddedUC} := \begin{pmatrix} 0\text{in} & C0(0\text{in}) \\ 5\text{in} & C5(0\text{in}) \\ 10\text{in} & C10(0\text{in}) \\ 15\text{in} & C15(0\text{in}) \\ 20\text{in} & C20(0\text{in}) \\ 25\text{in} & C25(0\text{in}) \\ 30\text{in} & C30(0\text{in}) \\ 35\text{in} & C35(0\text{in}) \end{pmatrix}$$

$$\text{AddedUCConstant} := \text{lspline}(\text{AddedUC}^{\langle 1 \rangle}, \text{AddedUC}^{\langle 2 \rangle})$$

$$\text{AddedUC}(X) := \text{interp}(\text{AddedUCConstant}, \text{AddedUC}^{\langle 1 \rangle}, \text{AddedUC}^{\langle 2 \rangle}, X)$$

$$\text{UCD} := 32\text{in}$$

$\text{UCD} := \text{root}(\text{AddedUC}(\text{UCD}) + Dr_1 - WTh, \text{UCD})$	UCD = 32.15 in
$\text{UCD} := \text{root}(\text{AddedUC}(\text{UCD}) + Dr_2 - WTh, \text{UCD})$	UCD = 28.713 in
$\text{UCD} := \text{root}(\text{AddedUC}(\text{UCD}) + Dr_3 - WTh, \text{UCD})$	UCD = 24.672 in
$\text{UCD} := \text{root}(\text{AddedUC}(\text{UCD}) + Dr_4 - WTh, \text{UCD})$	UCD = 19.53 in

VII.B. Specifying positions of each drilling depth at First Hui Depth_Huil

At First Hui

ID := 1.349cm

$ID := \text{root}(WTh - Dr_5 - CHuil(ID), ID)$	ID = 1.349 cm
$ID := \text{root}(WTh - Dr_6 - CHuil(ID), ID)$	ID = 3.376 cm
$ID := \text{root}(WTh - Dr_7 - CHuil(ID), ID)$	ID = 4.536 cm
$ID := \text{root}(WTh - Dr_8 - CHuil(ID), ID)$	ID = 5.419 cm
$ID := \text{root}(WTh - Dr_9 - CHuil(ID), ID)$	ID = 6.127 cm
$ID := \text{root}(WTh - Dr_{10} - CHuil(ID), ID)$	ID = 6.742 cm
$ID := \text{root}(WTh - Dr_{11} - CHuil(ID), ID)$	ID = 7.281 cm
$ID := \text{root}(WTh - Dr_{12} - CHuil(ID), ID)$	ID = 7.759 cm
$ID := \text{root}(WTh - Dr_{13} - CHuil(ID), ID)$	ID = 8.186 cm
$ID := \text{root}(WTh - Dr_{14} - CHuil(ID), ID)$	ID = 8.577 cm
$ID := \text{root}(WTh - Dr_{15} - CHuil(ID), ID)$	ID = 8.925 cm
$ID := \text{root}(WTh - Dr_{16} - CHuil(ID), ID)$	ID = 9.228 cm
$ID := \text{root}(WTh - Dr_{17} - CHuil(ID), ID)$	ID = 9.504 cm
$ID := \text{root}(WTh - Dr_{18} - CHuil(ID), ID)$	ID = 9.773 cm
$ID := \text{root}(WTh - Dr_{19} - CHuil(ID), ID)$	ID = 10.042 cm
$ID := \text{root}(WTh - Dr_{20} - CHuil(ID), ID)$	ID = 10.255 cm
$ID := \text{root}(WTh - Dr_{21} - CHuil(ID), ID)$	ID = 10.414 cm
$ID := \text{root}(WTh - Dr_{22} - CHuil(ID), ID)$	ID = 10.538 cm
$ID := \text{root}(WTh - Dr_{23} - CHuil(ID), ID)$	ID = 10.643 cm
$ID := \text{root}(WTh - Dr_{24} - CHuil(ID), ID)$	ID = 10.735 cm
$ID := \text{root}(WTh - Dr_{25} - CHuil(ID), ID)$	ID = 10.818 cm
$ID := \text{root}(WTh - Dr_{26} - CHuil(ID), ID)$	ID = 10.894 cm
$ID := \text{root}(WTh - Dr_{27} - CHuil(ID), ID)$	ID = 10.965 cm
$ID := \text{root}(WTh - Dr_{28} - CHuil(ID), ID)$	ID = 11.031 cm
$ID := \text{root}(WTh - Dr_{29} - CHuil(ID), ID)$	ID = 11.094 cm
$ID := \text{root}(WTh - Dr_{30} - CHuil(ID), ID)$	ID = 11.153 cm
$ID := \text{root}(WTh - Dr_{34} - CHuil(ID), ID)$	ID = 11.368 cm

VII.C. Specifying positions of each drilling depth from Bridge to First Hui

Depth_Bridge_to_Hui1

From Bridge to First Hui, in 1 inch intervals

5 in from bridge ID := 2.149cm

$\text{Dist}_7 - \text{Dist}_4 - 5\text{in} = 3.8\text{in}$

ID := root(WTh - Dr ₆ - CBH5(ID), ID)	ID = 2.153 cm
ID := root(WTh - Dr ₇ - CBH5(ID), ID)	ID = 3.746 cm
ID := root(WTh - Dr ₈ - CBH5(ID), ID)	ID = 4.835 cm
ID := root(WTh - Dr ₉ - CBH5(ID), ID)	ID = 5.663 cm
ID := root(WTh - Dr ₁₀ - CBH5(ID), ID)	ID = 6.338 cm
ID := root(WTh - Dr ₁₁ - CBH5(ID), ID)	ID = 6.931 cm
ID := root(WTh - Dr ₁₂ - CBH5(ID), ID)	ID = 7.453 cm
ID := root(WTh - Dr ₁₃ - CBH5(ID), ID)	ID = 7.916 cm
ID := root(WTh - Dr ₁₄ - CBH5(ID), ID)	ID = 8.333 cm
ID := root(WTh - Dr ₁₅ - CBH5(ID), ID)	ID = 8.715 cm
ID := root(WTh - Dr ₁₆ - CBH5(ID), ID)	ID = 9.052 cm
ID := root(WTh - Dr ₁₇ - CBH5(ID), ID)	ID = 9.346 cm
ID := root(WTh - Dr ₁₈ - CBH5(ID), ID)	ID = 9.619 cm
ID := root(WTh - Dr ₁₉ - CBH5(ID), ID)	ID = 9.892 cm
ID := root(WTh - Dr ₂₀ - CBH5(ID), ID)	ID = 10.147 cm
ID := root(WTh - Dr ₂₁ - CBH5(ID), ID)	ID = 10.347 cm
ID := root(WTh - Dr ₂₂ - CBH5(ID), ID)	ID = 10.495 cm
ID := root(WTh - Dr ₂₃ - CBH5(ID), ID)	ID = 10.613 cm
ID := root(WTh - Dr ₂₄ - CBH5(ID), ID)	ID = 10.715 cm
ID := root(WTh - Dr ₂₅ - CBH5(ID), ID)	ID = 10.804 cm
ID := root(WTh - Dr ₂₆ - CBH5(ID), ID)	ID = 10.885 cm
ID := root(WTh - Dr ₂₇ - CBH5(ID), ID)	ID = 10.96 cm
ID := root(WTh - Dr ₂₈ - CBH5(ID), ID)	ID = 11.03 cm
ID := root(WTh - Dr ₂₉ - CBH5(ID), ID)	ID = 11.095 cm
ID := root(WTh - Dr ₃₀ - CBH5(ID), ID)	ID = 11.157 cm
ID := root(WTh - Dr ₃₄ - CBH5(ID), ID)	ID = 11.378 cm

4 in from bridge ID := 0.73cm

$\text{Dist}_7 - \text{Dist}_4 - 4\text{in} = 4.8\text{in}$

$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_7 - \text{CBH4}(\text{ID}), \text{ID})$	ID = 0.73 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_8 - \text{CBH4}(\text{ID}), \text{ID})$	ID = 3.207 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_9 - \text{CBH4}(\text{ID}), \text{ID})$	ID = 4.455 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{10} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 5.382 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{11} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 6.111 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{12} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 6.741 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{13} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 7.294 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{14} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 7.784 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{15} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 8.222 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{16} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 8.621 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{17} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 8.982 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{18} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 9.297 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{19} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 9.578 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{20} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 9.849 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{21} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 10.122 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{22} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 10.351 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{23} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 10.521 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{24} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 10.653 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{25} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 10.761 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{26} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 10.856 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{27} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 10.941 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{28} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 11.019 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{29} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 11.091 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{30} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 11.158 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{34} - \text{CBH4}(\text{ID}), \text{ID})$	ID = 11.395 cm

3 in from bridge ID := 2.511cm

$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_9 - \text{CBH3}(\text{ID}), \text{ID})$	ID = 2.511 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{10} - \text{CBH3}(\text{ID}), \text{ID})$	ID = 4.005 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{11} - \text{CBH3}(\text{ID}), \text{ID})$	ID = 5.062 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{12} - \text{CBH3}(\text{ID}), \text{ID})$	ID = 5.874 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{13} - \text{CBH3}(\text{ID}), \text{ID})$	ID = 6.536 cm
$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{14} - \text{CBH3}(\text{ID}), \text{ID})$	ID = 7.121 cm

$ID := \text{root}(WTh - Dr_{15} - CBH3(ID), ID)$	ID = 7.64 cm
$ID := \text{root}(WTh - Dr_{16} - CBH3(ID), ID)$	ID = 8.102 cm
$ID := \text{root}(WTh - Dr_{17} - CBH3(ID), ID)$	ID = 8.518 cm
$ID := \text{root}(WTh - Dr_{18} - CBH3(ID), ID)$	ID = 8.9 cm
$ID := \text{root}(WTh - Dr_{19} - CBH3(ID), ID)$	ID = 9.238 cm
$ID := \text{root}(WTh - Dr_{20} - CBH3(ID), ID)$	ID = 9.534 cm
$ID := \text{root}(WTh - Dr_{21} - CBH3(ID), ID)$	ID = 9.808 cm
$ID := \text{root}(WTh - Dr_{22} - CBH3(ID), ID)$	ID = 10.082 cm
$ID := \text{root}(WTh - Dr_{23} - CBH3(ID), ID)$	ID = 10.338 cm
$ID := \text{root}(WTh - Dr_{24} - CBH3(ID), ID)$	ID = 10.538 cm
$ID := \text{root}(WTh - Dr_{25} - CBH3(ID), ID)$	ID = 10.686 cm
$ID := \text{root}(WTh - Dr_{26} - CBH3(ID), ID)$	ID = 10.804 cm
$ID := \text{root}(WTh - Dr_{27} - CBH3(ID), ID)$	ID = 10.906 cm
$ID := \text{root}(WTh - Dr_{28} - CBH3(ID), ID)$	ID = 10.995 cm
$ID := \text{root}(WTh - Dr_{29} - CBH3(ID), ID)$	ID = 11.076 cm
$ID := \text{root}(WTh - Dr_{30} - CBH3(ID), ID)$	ID = 11.151 cm
$ID := \text{root}(WTh - Dr_{34} - CBH3(ID), ID)$	ID = 11.408 cm

2 in from bridge ID := 1.467cm

$Dist_7 - Dist_4 - 2in = 6.8 in$

$ID := \text{root}(WTh - Dr_{10} - CBH2(ID), ID)$	ID = 1.467 cm
$ID := \text{root}(WTh - Dr_{11} - CBH2(ID), ID)$	ID = 3.491 cm
$ID := \text{root}(WTh - Dr_{12} - CBH2(ID), ID)$	ID = 4.692 cm
$ID := \text{root}(WTh - Dr_{13} - CBH2(ID), ID)$	ID = 5.601 cm
$ID := \text{root}(WTh - Dr_{14} - CBH2(ID), ID)$	ID = 6.315 cm
$ID := \text{root}(WTh - Dr_{15} - CBH2(ID), ID)$	ID = 6.934 cm
$ID := \text{root}(WTh - Dr_{16} - CBH2(ID), ID)$	ID = 7.482 cm
$ID := \text{root}(WTh - Dr_{17} - CBH2(ID), ID)$	ID = 7.97 cm
$ID := \text{root}(WTh - Dr_{18} - CBH2(ID), ID)$	ID = 8.407 cm
$ID := \text{root}(WTh - Dr_{19} - CBH2(ID), ID)$	ID = 8.806 cm
$ID := \text{root}(WTh - Dr_{20} - CBH2(ID), ID)$	ID = 9.168 cm
$ID := \text{root}(WTh - Dr_{21} - CBH2(ID), ID)$	ID = 9.484 cm
$ID := \text{root}(WTh - Dr_{22} - CBH2(ID), ID)$	ID = 9.766 cm
$ID := \text{root}(WTh - Dr_{23} - CBH2(ID), ID)$	ID = 10.039 cm
$ID := \text{root}(WTh - Dr_{24} - CBH2(ID), ID)$	ID = 10.313 cm

$ID := \text{root}(WTh - Dr_{25} - CBH2(ID), ID)$	ID = 10.542 cm
$ID := \text{root}(WTh - Dr_{26} - CBH2(ID), ID)$	ID = 10.712 cm
$ID := \text{root}(WTh - Dr_{27} - CBH2(ID), ID)$	ID = 10.844 cm
$ID := \text{root}(WTh - Dr_{28} - CBH2(ID), ID)$	ID = 10.952 cm
$ID := \text{root}(WTh - Dr_{29} - CBH2(ID), ID)$	ID = 11.047 cm
$ID := \text{root}(WTh - Dr_{30} - CBH2(ID), ID)$	ID = 11.132 cm
$ID := \text{root}(WTh - Dr_{34} - CBH2(ID), ID)$	ID = 11.414 cm

1 in from bridge ID := 2.838cm

$ID := \text{root}(WTh - Dr_{12} - CBH1(ID), ID)$	ID = 2.838 cm
$ID := \text{root}(WTh - Dr_{13} - CBH1(ID), ID)$	ID = 4.256 cm
$ID := \text{root}(WTh - Dr_{14} - CBH1(ID), ID)$	ID = 5.289 cm
$ID := \text{root}(WTh - Dr_{15} - CBH1(ID), ID)$	ID = 6.085 cm
$ID := \text{root}(WTh - Dr_{16} - CBH1(ID), ID)$	ID = 6.734 cm
$ID := \text{root}(WTh - Dr_{17} - CBH1(ID), ID)$	ID = 7.311 cm
$ID := \text{root}(WTh - Dr_{18} - CBH1(ID), ID)$	ID = 7.826 cm
$ID := \text{root}(WTh - Dr_{19} - CBH1(ID), ID)$	ID = 8.287 cm
$ID := \text{root}(WTh - Dr_{20} - CBH1(ID), ID)$	ID = 8.703 cm
$ID := \text{root}(WTh - Dr_{21} - CBH1(ID), ID)$	ID = 9.085 cm
$ID := \text{root}(WTh - Dr_{22} - CBH1(ID), ID)$	ID = 9.425 cm
$ID := \text{root}(WTh - Dr_{23} - CBH1(ID), ID)$	ID = 9.722 cm
$ID := \text{root}(WTh - Dr_{24} - CBH1(ID), ID)$	ID = 9.997 cm
$ID := \text{root}(WTh - Dr_{25} - CBH1(ID), ID)$	ID = 10.272 cm
$ID := \text{root}(WTh - Dr_{26} - CBH1(ID), ID)$	ID = 10.529 cm
$ID := \text{root}(WTh - Dr_{27} - CBH1(ID), ID)$	ID = 10.729 cm
$ID := \text{root}(WTh - Dr_{28} - CBH1(ID), ID)$	ID = 10.877 cm
$ID := \text{root}(WTh - Dr_{29} - CBH1(ID), ID)$	ID = 10.995 cm
$ID := \text{root}(WTh - Dr_{30} - CBH1(ID), ID)$	ID = 11.097 cm
$ID := \text{root}(WTh - Dr_{34} - CBH1(ID), ID)$	ID = 11.412 cm

VII.D. Specifying positions of each drilling depth at Bridge Depth_Bridge

At Bridge

ID := 1.96cm

ID := root(WTh - Dr ₁₃ - B_base(ID), ID)	ID = 1.96 cm
ID := root(WTh - Dr ₁₄ - B_base(ID), ID)	ID = 3.752 cm
ID := root(WTh - Dr ₁₅ - B_base(ID), ID)	ID = 4.928 cm
ID := root(WTh - Dr ₁₆ - B_base(ID), ID)	ID = 5.819 cm
ID := root(WTh - Dr ₁₇ - B_base(ID), ID)	ID = 6.518 cm
ID := root(WTh - Dr ₁₈ - B_base(ID), ID)	ID = 7.128 cm
ID := root(WTh - Dr ₁₉ - B_base(ID), ID)	ID = 7.67 cm
ID := root(WTh - Dr ₂₀ - B_base(ID), ID)	ID = 8.156 cm
ID := root(WTh - Dr ₂₁ - B_base(ID), ID)	ID = 8.592 cm
ID := root(WTh - Dr ₂₂ - B_base(ID), ID)	ID = 8.991 cm
ID := root(WTh - Dr ₂₃ - B_base(ID), ID)	ID = 9.354 cm
ID := root(WTh - Dr ₂₄ - B_base(ID), ID)	ID = 9.671 cm
ID := root(WTh - Dr ₂₅ - B_base(ID), ID)	ID = 9.955 cm
ID := root(WTh - Dr ₂₆ - B_base(ID), ID)	ID = 10.229 cm
ID := root(WTh - Dr ₂₇ - B_base(ID), ID)	ID = 10.504 cm
ID := root(WTh - Dr ₂₈ - B_base(ID), ID)	ID = 10.733 cm
ID := root(WTh - Dr ₂₉ - B_base(ID), ID)	ID = 10.903 cm
ID := root(WTh - Dr ₃₀ - B_base(ID), ID)	ID = 11.035 cm
ID := root(WTh - Dr ₃₄ - B_base(ID), ID)	ID = 11.402 cm

Finding where each line intersects the centerline (as measured from top of GuQin)

$$\text{Downslope} := \begin{pmatrix} \text{Dist}_4 - \text{Dist}_7 & \text{B_base}(0\text{in}) \\ \text{Dist}_4 + 1\text{in} & \text{CBH1}(0\text{in}) \\ \text{Dist}_4 + 2\text{in} & \text{CBH2}(0\text{in}) \\ \text{Dist}_4 + 3\text{in} & \text{CBH3}(0\text{in}) \\ \text{Dist}_4 + 4\text{in} & \text{CBH4}(0\text{in}) \\ \text{Dist}_4 + 5\text{in} & \text{CBH5}(0\text{in}) \\ \text{Dist}_7 - 3.3\text{in} & \text{CHui1}(0\text{in}) \end{pmatrix}$$

$$\text{DownslopeConstant} := \text{lspline}(\text{Downslope}^{(1)}, \text{Downslope}^{(2)})$$

$$\text{DS}(X) := \text{interp}(\text{DownslopeConstant}, \text{Downslope}^{(1)}, \text{Downslope}^{(2)}, X)$$

$$\text{DSD} := 43.27 \text{ cm}$$

$\text{DSD} := \text{root}(\text{DS}(\text{DSD}) + \text{Dr}_5 - \text{WTh}, \text{DSD})$	$\text{DSD} = 22.02 \text{ cm}$	$\text{DSD} - \text{Dist}_7 = -3.431 \text{ in}$
$\text{DSD} := \text{root}(\text{DS}(\text{DSD}) + \text{Dr}_6 - \text{WTh}, \text{DSD})$	$\text{DSD} = 20.234 \text{ cm}$	$\text{DSD} - \text{Dist}_7 = -4.134 \text{ in}$
$\text{DSD} := \text{root}(\text{DS}(\text{DSD}) + \text{Dr}_7 - \text{WTh}, \text{DSD})$	$\text{DSD} = 18.447 \text{ cm}$	$\text{DSD} - \text{Dist}_7 = -4.837 \text{ in}$
$\text{DSD} := \text{root}(\text{DS}(\text{DSD}) + \text{Dr}_8 - \text{WTh}, \text{DSD})$	$\text{DSD} = 16.664 \text{ cm}$	$\text{DSD} - \text{Dist}_7 = -5.539 \text{ in}$
$\text{DSD} := \text{root}(\text{DS}(\text{DSD}) + \text{Dr}_9 - \text{WTh}, \text{DSD})$	$\text{DSD} = 14.876 \text{ cm}$	$\text{DSD} - \text{Dist}_7 = -6.243 \text{ in}$
$\text{DSD} := \text{root}(\text{DS}(\text{DSD}) + \text{Dr}_{10} - \text{WTh}, \text{DSD})$	$\text{DSD} = 13.078 \text{ cm}$	$\text{DSD} - \text{Dist}_7 = -6.951 \text{ in}$
$\text{DSD} := \text{root}(\text{DS}(\text{DSD}) + \text{Dr}_{11} - \text{WTh}, \text{DSD})$	$\text{DSD} = 11.322 \text{ cm}$	$\text{DSD} - \text{Dist}_7 = -7.642 \text{ in}$
$\text{DSD} := \text{root}(\text{DS}(\text{DSD}) + \text{Dr}_{12} - \text{WTh}, \text{DSD})$	$\text{DSD} = 9.342 \text{ cm}$	$\text{DSD} - \text{Dist}_7 = -8.422 \text{ in}$

Bridge slot (like 36th line)

$$\text{ID} := 11.542 \text{ cm}$$

$$\text{ID} := \text{root}(\text{WTh} - \text{Dr}_{36} - \text{B_base}(\text{ID}), \text{ID}) \quad \text{ID} = 11.542 \text{ cm}$$

$$\text{B_base}(\text{ID}) = -0.458 \text{ cm}$$

VIII. Calculating the Hui Positioning

$$\text{String_length} := \text{Distance}(4, 12)$$

$$\text{Dist_Hui}_1 := \frac{1}{8} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_8 := \frac{3}{5} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_2 := \frac{1}{6} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_9 := \frac{4}{6} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_3 := \frac{1}{5} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_{10} := \frac{3}{4} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_4 := \frac{2}{8} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_{11} := \frac{4}{5} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_5 := \frac{2}{6} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_{12} := \frac{5}{6} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_6 := \frac{2}{5} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_{13} := \frac{7}{8} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui}_7 := \frac{1}{2} \cdot \text{String_length} + \text{Dist}_4$$

$$\text{Dist_Hui} = \begin{pmatrix} 22.35 \\ 27.01 \\ 30.73 \\ 36.32 \\ 45.64 \\ 53.09 \\ 64.26 \\ 75.44 \\ 82.89 \\ 92.20 \\ 97.79 \\ 101.52 \\ 106.17 \end{pmatrix} \text{ cm}$$

$$\text{Dist_Hui} = \begin{pmatrix} 7.2 \\ 8.7 \\ 9.9 \\ 11.7 \\ 14.7 \\ 17.1 \\ 20.7 \\ 24.3 \\ 26.7 \\ 29.7 \\ 31.5 \\ 32.7 \\ 34.2 \end{pmatrix} \text{ cun}$$

IX. Designing the Bottom Panel

IX.A. Calculating the positions of the Pool and Pond

Lines numbered 2, 3, 4, 5, 9, 11, and 12 correspond to that of the Top Panel

"In the middle between the fourth and seventh hui markers, measure off four cun up and down, altogether equal to eight cun, which stand for the eight winds. This is the length for the dragon pool."

$$\text{Dist_Mid_dragonpool} := \frac{(\text{Dist_Hui}_7 - \text{Dist_Hui}_4)}{2} + \text{Dist_Hui}_4$$

$$\text{Dist_Mid_dragonpool} = 50.292 \text{ cm}$$

$$4\text{cun} = 12.418 \text{ cm}$$

"In the middle of the tenth and thirteenth hui positions, measure off two inches up and down, altogether equal to 4 cun , which represents the four seasons. This is the length of the phoenix pond.

$$\text{Dist_Mid_phoenixpond} := \frac{(\text{Dist_Hui}_{13} - \text{Dist_Hui}_{10})}{2} + \text{Dist_Hui}_{10}$$

$$\text{Dist_Mid_phoenixpond} = 99.187 \text{ cm}$$

$$2\text{cun} = 6.209 \text{ cm}$$

"Now from the axis measure off $\frac{4}{10}$ cun for the width of the pool and of the pond, altogether $\frac{8}{10}$ cun to the left and right."

$$0.4\text{cun} = 1.242 \text{ cm}$$

$$0.8\text{cun} = 2.484 \text{ cm}$$

IX.B. Calculating the positions for the Wild Geese Feet holes

"At the center of the waist, away from the sides $\frac{3-4}{10}$ cun on both left and right, draw lines to make squares $\frac{1}{2}$ cun on a side for the holes for the wild geese feet."

$$0.35\text{cun} = 1.087 \text{ cm}$$

$$0.5\text{cun} = 1.552 \text{ cm}$$

IX.C. Calculating the position of the Peg Pool

"From the third crosswise line, measure up and down $\frac{4}{10}$ cun , altogether $\frac{8}{10}$ cun . Now from the baseline measure out on left and right $2 \frac{2}{10}$ cun , altogether $4 \frac{4}{10}$ cun .

$$0.4\text{cun} = 1.242 \text{ cm}$$

$$2.2\text{cun} = 6.83 \text{ cm}$$

IX.D. Determining the thickness of the Bottom Panel

Total Thickness	$0.6\text{cun} = 1.863 \text{ cm}$
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Demarcating line	$0.35\text{cun} = 1.087 \text{ cm}$
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Thickness of curves	$0.25\text{cun} = 0.776 \text{ cm}$
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X. Calculating the dimensions of the accessories

X.A. Bridge

Width	$0.3\text{cun} = 0.931\text{ cm}$
Set into face	$0.3\text{cun} = 0.931\text{ cm}$
Height above face	$0.45\text{cun} = 1.397\text{ cm}$

X.B. Nut

Height	$0.05\text{cun} = 0.155\text{ cm}$
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X.C. Receiving Dew

Thickness	$0.2\text{cun} = 0.621\text{ cm}$
Width	$0.4\text{cun} = 1.242\text{ cm}$

X.D. Peg Pool

Thickness	$0.2\text{cun} = 0.621\text{ cm}$
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X.E. Gums Supporter

Thickness	$0.25\text{cun} = 0.776\text{ cm}$
Breadth	$2\text{cun} = 6.209\text{ cm}$

X.F. Wild Duck Feet

Square	$1\text{cun} = 3.104\text{ cm}$
Height	$1.5\text{cun} = 4.657\text{ cm}$

X.G. Wild Geese Feet

Diameter	$1\text{cun} = 3.104\text{ cm}$
Height of round part	$0.5\text{cun} = 1.552\text{ cm}$
Square of post	$0.5\text{cun} = 1.552\text{ cm}$
Height of post	$1.25\text{cun} = 3.881\text{ cm}$
Distance embedded	$0.45\text{cun} = 1.397\text{ cm}$

XI.

Sound Posts

XI.A. Pillar of Heaven

Round diameter 0.3cun = 0.931 cm

$$\text{Position_Pillar_h} := \frac{(\text{Dist_Hui}_4 - \text{Dist_Hui}_3)}{2} + \text{Dist_Hui}_3$$

Position_Pillar_h = 10.8 cun

XI.B. Pillar of Earth

Square diameter 0.3cun = 0.931 cm

$$\text{Position_Pillar_e} := \frac{(\text{Dist_Hui}_8 - \text{Dist_Hui}_7)}{2} + \text{Dist_Hui}_7$$

Position_Pillar_e = 22.5 cun

