Specifications and Calculations
for GuQin Construction

I.   Defining chi and cun

$$\text{cun} \equiv \frac{11}{9} \cdot 2.54\text{cm} \qquad \text{chi} \equiv 10\text{cun} \qquad \text{ORIGIN} \equiv 1$$

II.   Determining the distance to each Line of Reference from the Baseline

Surface Dimensions:

12 Intervals between 13 Lines of Reference

$$\text{interval} := 1 .. 12 \qquad \text{lines} := 1 .. 13 \qquad \text{successive\_lines} := 2 .. 13$$

These are the distance from each Line of Reference to the next, between lines 1 through 13.

$$\text{Spacing}_{\text{interval}} :=$$

Line		Spacing
Line 01	**Baseline**	2cun
Line 02	**Forehead**	0.4cun
Line 03	**Receiving Dew**	0.3cun
Line 04	**Bridge**	0.8cun
Line 05	**Beginning of Nape**	3.2cun
Line 06	**Center of Neck**	3.2cun
Line 07	**Shoulder**	1chi + 4.4cun
Line 08	**Beginning of Waist**	3.6cun
Line 09	**Center of Waist**	3.6cun
Line 10	**End of Waist**	6.2cun
Line 11	**Cap**	1cun
Line 12	**Tail-end**	0.3cun
Line 13	**Mouth of Nut**	

Calculation of Cumulative Distances from Baseline to Lines of Reference

$$\text{Distance}(\text{begin}, \text{end}) := \sum_{i\,=\,\text{begin}}^{\text{end}-1} \text{Spacing}_i$$

$$\text{Dist}_{\text{successive\_lines}} := \text{Distance}(1, \text{successive\_lines})$$

Distances between Baseline to Lines of Reference in cun , inches, and centimeters

$$\text{lines} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \end{pmatrix} \quad \text{Dist} = \begin{pmatrix} 0 \\ 2 \\ 2.4 \\ 2.7 \\ 3.5 \\ 6.7 \\ 9.9 \\ 24.3 \\ 27.9 \\ 31.5 \\ 37.7 \\ 38.7 \\ 39 \end{pmatrix} \text{cun} \quad \text{Dist} = \begin{pmatrix} 0.00 \\ 2.44 \\ 2.93 \\ 3.30 \\ 4.28 \\ 8.19 \\ 12.10 \\ 29.70 \\ 34.10 \\ 38.50 \\ 46.08 \\ 47.30 \\ 47.67 \end{pmatrix} \text{in} \quad \text{Dist} = \begin{pmatrix} 0 \\ 6.209 \\ 7.451 \\ 8.382 \\ 10.866 \\ 20.8 \\ 30.734 \\ 75.438 \\ 86.614 \\ 97.79 \\ 117.038 \\ 120.142 \\ 121.073 \end{pmatrix} \text{cm}$$

III.   Calculating the width of the GuQin at each Line of Reference

Width of GuQin from Axis Line to Edge with respect to each Line of Reference

$$\text{Width}_1 := 5.6\,\text{cun}$$

$$\text{Width}_5 := 6\,\text{cun}$$

Line 01 Baseline
Line 02 Forehead
Line 03 Receiving Dew
Line 04 Bridge
Line 05 Beginning of Nape
Line 06 Center of Neck
Line 07 Shoulder
Line 08 Beginning of Waist
Line 09 Center of Waist
Line 10 End of Waist
Line 11 Cap
Line 12 Tail-end
Line 13 Mouth of Nut

$$\text{Width}_2 := 2 \cdot \left[\frac{\left(\dfrac{\text{Width}_5}{2} - \dfrac{\text{Width}_1}{2} \right)}{\text{Distance}(1,5)} \cdot \text{Distance}(1,2) + \frac{\text{Width}_1}{2} \right] + 1 \cdot \text{cun}$$

$$\text{Width}_4 := 2 \cdot \left[\frac{\left(\dfrac{\text{Width}_5}{2} - \dfrac{\text{Width}_1}{2} \right)}{\text{Distance}(1,5)} \cdot \text{Distance}(1,4) + \frac{\text{Width}_1}{2} \right] + 1 \cdot \text{cun}$$

$$\text{Width}_6 := 5.6\,\text{cun}$$

$$\text{Width}_7 := 7 \cdot \text{cun}$$

$$\text{Width}_{11} := 5\,\text{cun}$$

$$\text{Width}_9 := 2 \cdot \left[\frac{\left(\dfrac{\text{Width}_{11}}{2} - \dfrac{\text{Width}_7}{2} \right)}{\text{Distance}(7,11)} \cdot \text{Distance}(7,9) + \frac{\text{Width}_7}{2} \right] - 0.3\text{cun}$$

$$\text{Width}_{12} := 2 \cdot \left[\frac{\left(\dfrac{\text{Width}_{11}}{2} - \dfrac{\text{Width}_7}{2} \right)}{\text{Distance}(7,11)} \cdot \text{Distance}(7,12) + \frac{\text{Width}_7}{2} \right]$$

$$\text{Width}_{13} := 1.2\text{cun}$$

$$\text{Half\_Width} := \frac{\text{Width}}{2}$$

$\text{Half\_Width}_1 = 2.8\,\text{cun}$

$\text{Half\_Width}_2 = 3.414\,\text{cun}$

$\text{Half\_Width}_4 = 3.454\,\text{cun}$

$\text{Half\_Width}_5 = 3\,\text{cun}$

$\text{Half\_Width}_6 = 2.8\,\text{cun}$

$\text{Half\_Width}_7 = 3.5\,\text{cun}$

$\text{Half\_Width}_9 = 2.703\,\text{cun}$

$\text{Half\_Width}_{11} = 2.5\,\text{cun}$

$\text{Half\_Width}_{12} = 2.464\,\text{cun}$

$\text{Half\_Width}_{13} = 0.6\,\text{cun}$

$$\text{Top\_View} := \begin{pmatrix} \text{Dist}_1 & \text{Half\_Width}_1 \\ \text{Dist}_2 & \text{Half\_Width}_2 \\ \text{Dist}_4 & \text{Half\_Width}_4 \\ \text{Dist}_5 & \text{Half\_Width}_5 \\ \text{Dist}_6 & \text{Half\_Width}_6 \\ \text{Dist}_7 & \text{Half\_Width}_7 \\ \text{Dist}_9 & \text{Half\_Width}_9 \\ \text{Dist}_{11} & \text{Half\_Width}_{11} \\ \text{Dist}_{13} & \text{Half\_Width}_{13} \end{pmatrix} \qquad \text{Top\_View} = \begin{pmatrix} 0 & 8.692 \\ 6.209 & 10.599 \\ 8.382 & 10.724 \\ 10.866 & 9.313 \\ 20.8 & 8.692 \\ 30.734 & 10.866 \\ 86.614 & 8.39 \\ 117.038 & 7.761 \\ 121.073 & 1.863 \end{pmatrix} \text{cm}$$

$$\frac{\text{Top\_View}^{\langle 2 \rangle}}{\text{cun}}$$

×××

$$\frac{-\ \text{Top\_View}^{\langle 2 \rangle}}{\text{cun}}$$

×××

$$\frac{\text{Top\_View}^{\langle 1 \rangle}}{\text{cun}}$$
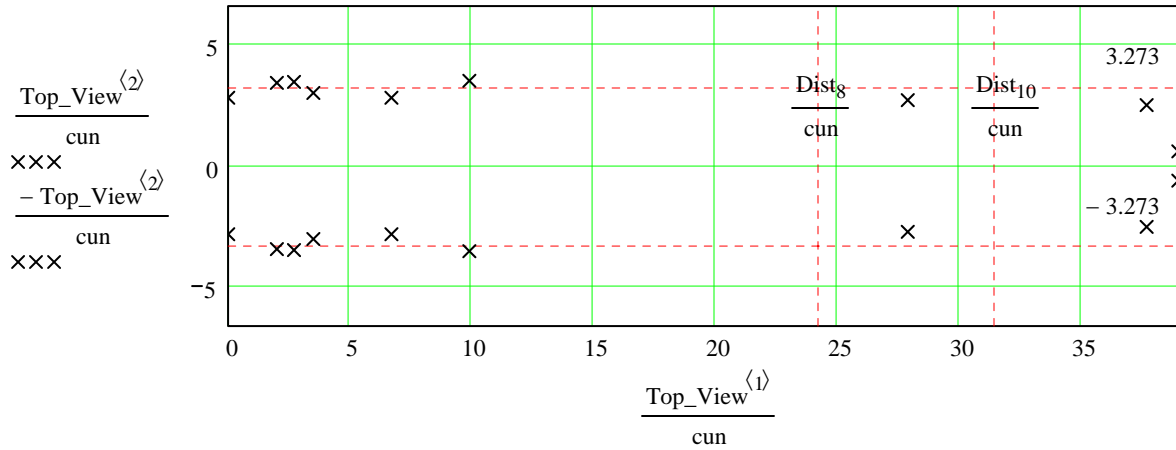
IV.   Developing the foundation for curvature calculation

IV.A.   Setting the Reference curvatures at the Shoulder and Nut

Calculations for Surface Contours

$$\text{Height\_c} := 0.65\,\text{cun}$$

$$\text{Surface}(HD, w, h) := \sqrt{\left[1 - \frac{HD^2}{\left(\dfrac{w}{2}\right)^2} \right] \cdot h^2}$$

Cross Section at Line 07: Shoulder

$$\frac{\text{Surface}\big(HD, \text{Width}_7, \text{Height\_c}\big)}{\text{cun}}$$

$$\frac{HD}{\text{cun}}$$

Cross Section at Line 11: Cap

$$\frac{\text{Surface}\big(HD, \text{Width}_{11}, \text{Height\_c}\big)}{\text{cun}}$$

$$\frac{HD}{\text{cun}}$$
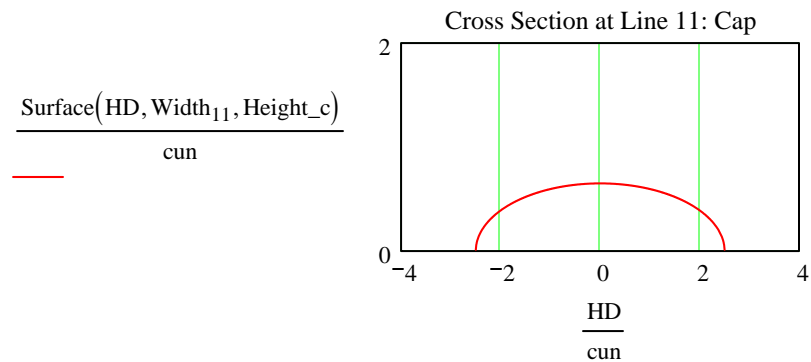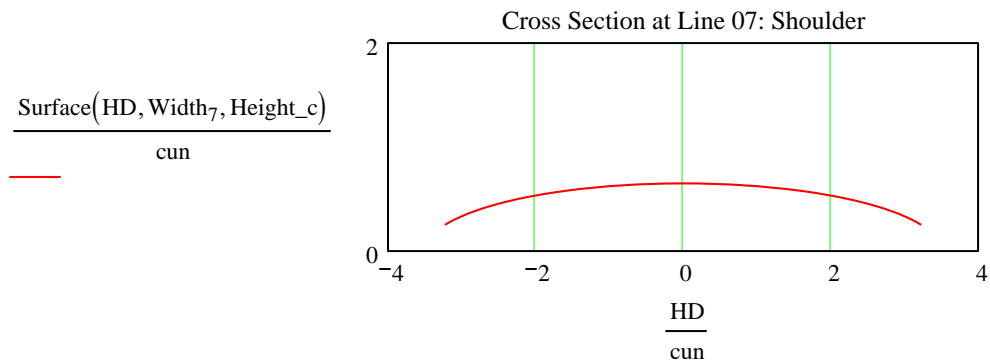
Calculating Longitudinal Slope Along Each String

String := 1 .. 7

$$\text{Shoulder\_String\_Width} := 2 \cdot \left[\frac{(0.55\text{cun} - 2\text{cun})}{\text{Distance}(4,12)} \cdot \text{Distance}(4,7) + 2\text{cun} \right]$$

Bridge_String_Width := 4cun

Shoulder_String_Width = 3.42 cun

Nut_String_Width := 1.1cun

$$\text{Shoulder\_String\_X\_Position}_{\text{String}} := (\text{String} - 4) \cdot \frac{\text{Shoulder\_String\_Width}}{6}$$

$$\text{Nut\_String\_X\_Position}_{\text{String}} := (\text{String} - 4) \cdot \frac{\text{Nut\_String\_Width}}{6}$$

$$\text{Shoulder\_String\_X\_Position} = \begin{pmatrix} -1.71 \\ -1.14 \\ -0.57 \\ 0 \\ 0.57 \\ 1.14 \\ 1.71 \end{pmatrix} \text{cun} \qquad \text{Nut\_String\_X\_Position} = \begin{pmatrix} -0.55 \\ -0.367 \\ -0.183 \\ 0 \\ 0.183 \\ 0.367 \\ 0.55 \end{pmatrix} \text{cun}$$

IV.B. Extracting points of the curvature at the Shoulder and Nut for the purpose of interpolation

$$N\_X1 := \frac{-\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 8 - 0.55cun$$

$$N\_X2 := \frac{-\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 7 - 0.55cun$$

$$N\_X3 := \frac{-\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 6 - 0.55cun$$

$$N\_X4 := \frac{-\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 5 - 0.55cun$$

$$N\_X5 := \frac{-\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 4 - 0.55cun$$

$$N\_X6 := \frac{-\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 3 - 0.55cun$$

$$N\_X7 := \frac{-\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 2 - 0.55cun$$

$$N\_X8 := \frac{-\left(Half\_Width_{12} - 0.55cun\right)}{8} - 0.55cun$$

$$N\_X9 := -(0.55cun)$$

$$N\_X10 := Nut\_String\_X\_Position_2$$

$$N\_X11 := Nut\_String\_X\_Position_3$$

$$N\_X12 := Nut\_String\_X\_Position_4$$

$$N\_X13 := Nut\_String\_X\_Position_5$$

$$N\_X14 := Nut\_String\_X\_Position_6$$

$$N\_X15 := 0.55cun$$

$$N\_X16 := \frac{Half\_Width_{12} - 0.55cun}{8} + 0.55cun$$

$$N\_X17 := \frac{\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 2 + 0.55cun$$

$$N\_X18 := \frac{\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 3 + 0.55cun$$

$$N\_X19 := \frac{\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 4 + 0.55cun$$

$$N\_X20 := \frac{\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 5 + 0.55cun$$

$$N\_X21 := \frac{\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 6 + 0.55cun$$

$$N\_X22 := \frac{\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 7 + 0.55cun$$

$$N\_X23 := \frac{\left(Half\_Width_{12} - 0.55cun\right)}{8} \cdot 8 + 0.55cun$$

$$N\_XY := \begin{pmatrix} N\_X1 & Surface\left(N\_X1, Width_{12}, Height\_c\right) \\ N\_X2 & Surface\left(N\_X2, Width_{12}, Height\_c\right) \\ N\_X3 & Surface\left(N\_X3, Width_{12}, Height\_c\right) \\ N\_X4 & Surface\left(N\_X4, Width_{12}, Height\_c\right) \\ N\_X5 & Surface\left(N\_X5, Width_{12}, Height\_c\right) \\ N\_X6 & Surface\left(N\_X6, Width_{12}, Height\_c\right) \\ N\_X7 & Surface\left(N\_X7, Width_{12}, Height\_c\right) \\ N\_X8 & Surface\left(N\_X8, Width_{12}, Height\_c\right) \\ N\_X9 & Surface\left(N\_X9, Width_{12}, Height\_c\right) \\ N\_X10 & Surface\left(N\_X10, Width_{12}, Height\_c\right) \\ N\_X11 & Surface\left(N\_X11, Width_{12}, Height\_c\right) \\ N\_X12 & Surface\left(N\_X12, Width_{12}, Height\_c\right) \\ N\_X13 & Surface\left(N\_X13, Width_{12}, Height\_c\right) \\ N\_X14 & Surface\left(N\_X14, Width_{12}, Height\_c\right) \\ N\_X15 & Surface\left(N\_X15, Width_{12}, Height\_c\right) \\ N\_X16 & Surface\left(N\_X16, Width_{12}, Height\_c\right) \\ N\_X17 & Surface\left(N\_X17, Width_{12}, Height\_c\right) \\ N\_X18 & Surface\left(N\_X18, Width_{12}, Height\_c\right) \\ N\_X19 & Surface\left(N\_X19, Width_{12}, Height\_c\right) \\ N\_X20 & Surface\left(N\_X20, Width_{12}, Height\_c\right) \\ N\_X21 & Surface\left(N\_X21, Width_{12}, Height\_c\right) \\ N\_X22 & Surface\left(N\_X22, Width_{12}, Height\_c\right) \\ N\_X23 & Surface\left(N\_X23, Width_{12}, Height\_c\right) \end{pmatrix}$$

$$N\_XY = \begin{pmatrix} -2.464 & 0 \\ -2.225 & 0.279 \\ -1.986 & 0.385 \\ -1.746 & 0.459 \\ -1.507 & 0.514 \\ -1.268 & 0.557 \\ -1.029 & 0.591 \\ -0.789 & 0.616 \\ -0.55 & 0.634 \\ -0.367 & 0.643 \\ -0.183 & 0.648 \\ 0 & 0.65 \\ 0.183 & 0.648 \\ 0.367 & 0.643 \\ 0.55 & 0.634 \\ 0.789 & 0.616 \\ 1.029 & 0.591 \\ 1.268 & 0.557 \\ 1.507 & 0.514 \\ 1.746 & 0.459 \\ 1.986 & 0.385 \\ 2.225 & 0.279 \\ 2.464 & 0 \end{pmatrix} cun$$

$$S\_X1 := \frac{-\left(\text{Half\_Width}_7 + \text{Shoulder\_String\_X\_Position}_1\right)}{8} \cdot 8 + \text{Shoulder\_String\_X\_Position}_1$$

$$S\_X2 := \frac{-\left(\text{Half\_Width}_7 + \text{Shoulder\_String\_X\_Position}_1\right)}{8} \cdot 7 + \text{Shoulder\_String\_X\_Position}_1$$

$$S\_X3 := \frac{-\left(\text{Half\_Width}_7 + \text{Shoulder\_String\_X\_Position}_1\right)}{8} \cdot 6 + \text{Shoulder\_String\_X\_Position}_1$$

$$S\_X4 := \frac{-\left(\text{Half\_Width}_7 + \text{Shoulder\_String\_X\_Position}_1\right)}{8} \cdot 5 + \text{Shoulder\_String\_X\_Position}_1$$

$$S\_X5 := \frac{-\left(\text{Half\_Width}_7 + \text{Shoulder\_String\_X\_Position}_1\right)}{8} \cdot 4 + \text{Shoulder\_String\_X\_Position}_1$$

$$S\_X6 := \frac{-\left(\text{Half\_Width}_7 + \text{Shoulder\_String\_X\_Position}_1\right)}{8} \cdot 3 + \text{Shoulder\_String\_X\_Position}_1$$

$$S\_X7 := \frac{-\left(\text{Half\_Width}_7 + \text{Shoulder\_String\_X\_Position}_1\right)}{8} \cdot 2 + \text{Shoulder\_String\_X\_Position}_1$$

$$S\_X8 := \frac{-\left(\text{Half\_Width}_7 + \text{Shoulder\_String\_X\_Position}_1\right)}{8} + \text{Shoulder\_String\_X\_Position}_1$$

$$S\_X9 := \text{Shoulder\_String\_X\_Position}_1$$

$$S\_X10 := \text{Shoulder\_String\_X\_Position}_2$$

$$S\_X11 := \text{Shoulder\_String\_X\_Position}_3$$

$$S\_X12 := \text{Shoulder\_String\_X\_Position}_4$$

$$S\_X13 := \text{Shoulder\_String\_X\_Position}_5$$

$$S\_X14 := \text{Shoulder\_String\_X\_Position}_6$$

$$S\_X15 := \text{Shoulder\_String\_X\_Position}_7$$

$$S\_X16 := \frac{\left(\text{Half\_Width}_7 - \text{Shoulder\_String\_X\_Position}_7\right)}{8} + \text{Shoulder\_String\_X\_Position}_7$$

$$S\_X17 := \frac{\left(\text{Half\_Width}_7 - \text{Shoulder\_String\_X\_Position}_7\right)}{8} \cdot 2 + \text{Shoulder\_String\_X\_Position}_7$$

$$S\_X18 := \frac{\left(\text{Half\_Width}_7 - \text{Shoulder\_String\_X\_Position}_7\right)}{8} \cdot 3 + \text{Shoulder\_String\_X\_Position}_7$$

$$S\_X19 := \frac{\left(\text{Half\_Width}_7 - \text{Shoulder\_String\_X\_Position}_7\right)}{8} \cdot 4 + \text{Shoulder\_String\_X\_Position}_7$$

$$S\_X20 := \frac{\left(\text{Half\_Width}_7 - \text{Shoulder\_String\_X\_Position}_7\right)}{8} \cdot 5 + \text{Shoulder\_String\_X\_Position}_7$$

$$S\_X21 := \frac{\left(\text{Half\_Width}_7 - \text{Shoulder\_String\_X\_Position}_7\right)}{8} \cdot 6 + \text{Shoulder\_String\_X\_Position}_7$$

$$S\_X22 := \frac{\left(\text{Half\_Width}_7 - \text{Shoulder\_String\_X\_Position}_7\right)}{8} \cdot 7 + \text{Shoulder\_String\_X\_Position}_7$$

$$S\_X23 := \frac{\left(\text{Half\_Width}_7 - \text{Shoulder\_String\_X\_Position}_7\right)}{8} \cdot 8 + \text{Shoulder\_String\_X\_Position}_7$$

$$S\_XY := \begin{pmatrix} S\_X1 & \text{Surface}\left(S\_X1, \text{Width}_7, \text{Height\_c}\right) \\ S\_X2 & \text{Surface}\left(S\_X2, \text{Width}_7, \text{Height\_c}\right) \\ S\_X3 & \text{Surface}\left(S\_X3, \text{Width}_7, \text{Height\_c}\right) \\ S\_X4 & \text{Surface}\left(S\_X4, \text{Width}_7, \text{Height\_c}\right) \\ S\_X5 & \text{Surface}\left(S\_X5, \text{Width}_7, \text{Height\_c}\right) \\ S\_X6 & \text{Surface}\left(S\_X6, \text{Width}_7, \text{Height\_c}\right) \\ S\_X7 & \text{Surface}\left(S\_X7, \text{Width}_7, \text{Height\_c}\right) \\ S\_X8 & \text{Surface}\left(S\_X8, \text{Width}_7, \text{Height\_c}\right) \\ S\_X9 & \text{Surface}\left(S\_X9, \text{Width}_7, \text{Height\_c}\right) \\ S\_X10 & \text{Surface}\left(S\_X10, \text{Width}_7, \text{Height\_c}\right) \\ S\_X11 & \text{Surface}\left(S\_X11, \text{Width}_7, \text{Height\_c}\right) \\ S\_X12 & \text{Surface}\left(S\_X12, \text{Width}_7, \text{Height\_c}\right) \\ S\_X13 & \text{Surface}\left(S\_X13, \text{Width}_7, \text{Height\_c}\right) \\ S\_X14 & \text{Surface}\left(S\_X14, \text{Width}_7, \text{Height\_c}\right) \\ S\_X15 & \text{Surface}\left(S\_X15, \text{Width}_7, \text{Height\_c}\right) \\ S\_X16 & \text{Surface}\left(S\_X16, \text{Width}_7, \text{Height\_c}\right) \\ S\_X17 & \text{Surface}\left(S\_X17, \text{Width}_7, \text{Height\_c}\right) \\ S\_X18 & \text{Surface}\left(S\_X18, \text{Width}_7, \text{Height\_c}\right) \\ S\_X19 & \text{Surface}\left(S\_X19, \text{Width}_7, \text{Height\_c}\right) \\ S\_X20 & \text{Surface}\left(S\_X20, \text{Width}_7, \text{Height\_c}\right) \\ S\_X21 & \text{Surface}\left(S\_X21, \text{Width}_7, \text{Height\_c}\right) \\ S\_X22 & \text{Surface}\left(S\_X22, \text{Width}_7, \text{Height\_c}\right) \\ S\_X23 & \text{Surface}\left(S\_X23, \text{Width}_7, \text{Height\_c}\right) \end{pmatrix}$$

$$S\_XY = \begin{pmatrix} -3.5 & 0 \\ -3.276 & 0.229 \\ -3.052 & 0.318 \\ -2.829 & 0.383 \\ -2.605 & 0.434 \\ -2.381 & 0.476 \\ -2.157 & 0.512 \\ -1.934 & 0.542 \\ -1.71 & 0.567 \\ -1.14 & 0.615 \\ -0.57 & 0.641 \\ 0 & 0.65 \\ 0.57 & 0.641 \\ 1.14 & 0.615 \\ 1.71 & 0.567 \\ 1.934 & 0.542 \\ 2.157 & 0.512 \\ 2.381 & 0.476 \\ 2.605 & 0.434 \\ 2.829 & 0.383 \\ 3.052 & 0.318 \\ 3.276 & 0.229 \\ 3.5 & 0 \end{pmatrix} \text{cun}$$

V.   Calculating Bridge geometry

V.A.   Interpolating data points at the Bridge, using the Reference curvatures at the Shoulder and Nut

$$B\_XY := \frac{(S\_XY - N\_XY)}{\text{Distance}(7,12)} \cdot \text{Distance}(4,12) + N\_XY$$

$$B\_X := B\_XY^{\langle 1 \rangle}$$

$$B\_Y := B\_XY^{\langle 2 \rangle}$$

$$B\_XY = \begin{pmatrix} -3.759 & 0 \\ -3.539 & 0.216 \\ -3.319 & 0.301 \\ -3.099 & 0.364 \\ -2.879 & 0.414 \\ -2.66 & 0.456 \\ -2.44 & 0.492 \\ -2.22 & 0.523 \\ -2 & 0.551 \\ -1.333 & 0.608 \\ -0.667 & 0.64 \\ 0 & 0.65 \\ 0.667 & 0.64 \\ 1.333 & 0.608 \\ 2 & 0.551 \\ 2.22 & 0.523 \\ 2.44 & 0.492 \\ 2.66 & 0.456 \\ 2.879 & 0.414 \\ 3.099 & 0.364 \\ 3.319 & 0.301 \\ 3.539 & 0.216 \\ 3.759 & 0 \end{pmatrix} \text{cun}$$

V.B.   Deriving the Function B_Y_interp(x) describing the curvature of the interpolated data points

$$B\_S := \text{pspline}(B\_X, B\_Y) \qquad \text{Spline vector constant}$$

$$B\_Y\_interp(X) := \text{interp}(B\_S, B\_X, B\_Y, X) \qquad \text{Curvature at bridge from interpolation of the face}$$
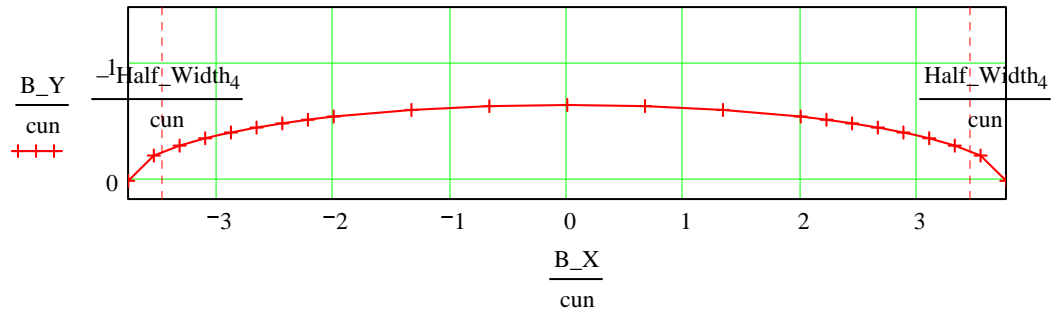
V.C. **Deriving the function B_height(x) describing the curvature of the top of the Bridge**

B_height(X) := B_Y_interp(X) + 0.25cun

V.D. **Deriving the function B_base(x) describing the curvature of the bottom of the Bridge**

B_base(X) := B_Y_interp(X) − 0.2cun
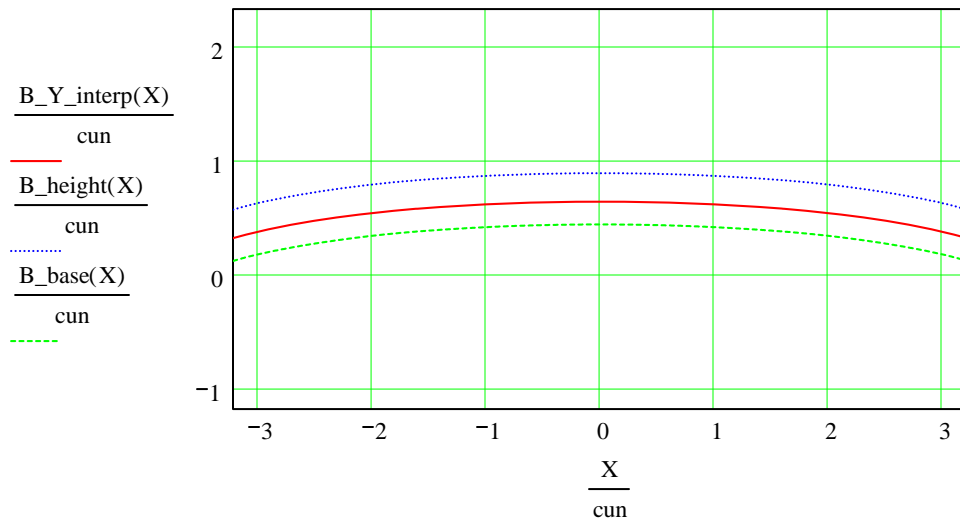
V.E.   **Ensuring equal spacing of Strings at the Bridge**

                 V.E.1.   **Calculation of String spacing measured along the curvature of the top of the bridge**

B_sw = The length of curvature along the top of the bridge, from string 1 to string 7.
B_sp = The distance of each String Position measured along the curvature of the top of the bridge.

$$B\_sw := \int_{-\frac{Bridge\_String\_Width}{2}}^{\frac{Bridge\_String\_Width}{2}} \sqrt{1 + \left(\frac{d}{dX}B\_height(X)\right)^2}\, dX$$

$$B\_sw = 4.007 \text{ cun}$$

$$B\_sp_{String} := (String - 4)\cdot\frac{B\_sw}{6}$$

$$B\_sp = \begin{pmatrix} -2.003 \\ -1.336 \\ -0.668 \\ 0 \\ 0.668 \\ 1.336 \\ 2.003 \end{pmatrix} \text{cun}$$

X Positon of Each String at Bridge Top

B_sX_bt = The X coordinate of each string while seated at bridge top, measured horizontally from the center axis of the GuQin , while B_sY_bt = the Y coordinate.

         For Strings 1 and 7

             $x\_dist := 2\text{cun}$

             Given

$$B\_sp_7 = \int_{0\text{cun}}^{x\_dist} \sqrt{1 + \left(\frac{d}{dX}B\_height(X)\right)^2}\, dX$$

$$B\_sX\_bt_7 := Find(x\_dist)$$

$$B\_sX\_bt_1 := -B\_sX\_bt_7$$

For Strings 2 and 6

$$\text{x\_dist} := 1.3 \text{cun}$$

Given

$$B\_sp_6 = \int_{0\text{cun}}^{\text{x\_dist}} \sqrt{\left[1 + \left(\frac{d}{dX} B\_height(X)\right)^2\right]} \, dX$$

$$B\_sX\_bt_6 := \text{Find}(\text{x\_dist})$$

$$B\_sX\_bt_2 := -B\_sX\_bt_6$$

For Strings 3 and 5

$$\text{x\_dist} := 0.7 \text{cun}$$

Given

$$B\_sp_5 = \int_{0\text{cun}}^{\text{x\_dist}} \sqrt{\left[1 + \left(\frac{d}{dX} B\_height(X)\right)^2\right]} \, dX$$

$$B\_sX\_bt_5 := \text{Find}(\text{x\_dist})$$

$$B\_sX\_bt_3 := -B\_sX\_bt_5$$

$$B\_sY\_bt_{\text{String}} := B\_height\left(B\_sX\_bt_{\text{String}}\right)$$

$$B\_S\_bt^{\langle 1 \rangle} := B\_sX\_bt$$

$$B\_S\_bt^{\langle 2 \rangle} := B\_sY\_bt$$

These are the X,Y coordinates of each string at the height of the bridge.

$$B\_S\_bt = \begin{pmatrix} -2 & 0.801 \\ -1.335 & 0.857 \\ -0.668 & 0.89 \\ 0 & 0.9 \\ 0.668 & 0.89 \\ 1.335 & 0.857 \\ 2 & 0.801 \end{pmatrix} \text{cun}$$

These are the distance from the center axis line along the top of the bridge for each string.

$$B\_sp = \begin{pmatrix} -2.003 \\ -1.336 \\ -0.668 \\ 0 \\ 0.668 \\ 1.336 \\ 2.003 \end{pmatrix} cun$$

V.E.2. Calculation of Tuning Hole positioning

V.E.2.a. Finding the XY position of eachTuning Hole

Calculation of the Slope of the Bridge at Each String Position

$$xx := B\_sX\_bt_1$$

$$Slope_1 := \frac{d}{dxx} B\_height(xx)$$

$$xx := B\_sX\_bt_2$$

$$Slope_2 := \frac{d}{dxx} B\_height(xx)$$

$$xx := B\_sX\_bt_3$$

$$Slope_3 := \frac{d}{dxx} B\_height(xx)$$

$$Slope_7 := -Slope_1$$

$$xx := B\_sX\_bt_4$$

$$Slope_6 := -Slope_2$$

$$Slope_4 := \frac{d}{dxx} B\_height(xx)$$

$$Slope_5 := -Slope_3$$

iSlope = inverse of Slope. This depicts the slope of the tuning cords, running perpendicular to the curvature of the bridge.

$$iSlope_1 := \frac{-1}{Slope_1} \qquad iSlope_5 := \frac{-1}{Slope_5}$$

$$\text{iSlope}_2 := \frac{-1}{\text{Slope}_2} \qquad \text{iSlope}_6 := \frac{-1}{\text{Slope}_6} \qquad \text{iSlope}_4 := \infty$$

$$\text{iSlope}_3 := \frac{-1}{\text{Slope}_3} \qquad \text{iSlope}_7 := \frac{-1}{\text{Slope}_7}$$

$$\text{Slope} = \begin{pmatrix} 0.114 \\ 0.064 \\ 0.032 \\ 0 \\ -0.032 \\ -0.064 \\ -0.114 \end{pmatrix} \qquad \text{iSlope} = \begin{pmatrix} -8.803 \\ -15.646 \\ -31.349 \\ 1 \times 10^{307} \\ 31.349 \\ 15.646 \\ 8.803 \end{pmatrix}$$

Finding the X,Y coordinates of the intersection between the Tuning Cords and the face of the GuQin , after adjusting for the 2/10 cun lowering of the face.

For Strings 1 and 7

$x := 2\text{cun} \qquad y := 0.3\text{cun}$

$i := 1 .. 2$

Given

$y = \text{B\_base}(x)$

$y = \text{iSlope}_7 \cdot x + \left(\text{B\_sY\_bt}_7 - \text{iSlope}_7 \cdot \text{B\_sX\_bt}_7 \right)$

$\text{B\_S\_f7} := \text{Find}(x, y)$

$\text{B\_S\_f1}_i :=$

$-\text{B\_S\_f7}_1$
B_S_f7_2

For Strings 2 and 6

Given

$y = \text{B\_base}(x)$

$y = \text{iSlope}_6 \cdot x + \left(\text{B\_sY\_bt}_6 - \text{iSlope}_6 \cdot \text{B\_sX\_bt}_6 \right)$

$\text{B\_S\_f6} := \text{Find}(x, y)$

$$\text{B\_S\_f2}_i :=$$

$$\begin{array}{|c|}\hline -\text{B\_S\_f6}_1 \\\hline \text{B\_S\_f6}_2 \\\hline\end{array}$$

For Strings 3 and 5

Given

$$y = \text{B\_base}(x)$$

$$y = \text{iSlope}_5 \cdot x + \left(\text{B\_sY\_bt}_5 - \text{iSlope}_5 \cdot \text{B\_sX\_bt}_5\right)$$

$$\text{B\_S\_f5} := \text{Find}(x, y)$$

$$\text{B\_S\_f3}_i :=$$

$$\begin{array}{|c|}\hline -\text{B\_S\_f5}_1 \\\hline \text{B\_S\_f5}_2 \\\hline\end{array}$$

For String 4

$$\text{B\_S\_f4}_i :=$$

$$\begin{array}{|c|}\hline \text{0cun} \\\hline \text{B\_base(0cun)} \\\hline\end{array}$$

The X,Y coordinates of the tuning cord and face intersection definition:

$$\text{tB\_S\_f}^{\langle 1 \rangle} := \text{B\_S\_f1} \qquad \text{tB\_S\_f}^{\langle 5 \rangle} := \text{B\_S\_f5}$$

$$\text{tB\_S\_f}^{\langle 2 \rangle} := \text{B\_S\_f2} \qquad \text{tB\_S\_f}^{\langle 6 \rangle} := \text{B\_S\_f6}$$

$$\text{tB\_S\_f}^{\langle 3 \rangle} := \text{B\_S\_f3} \qquad \text{tB\_S\_f}^{\langle 7 \rangle} := \text{B\_S\_f7}$$

$$\text{tB\_S\_f}^{\langle 4 \rangle} := \text{B\_S\_f4}$$

$$\text{B\_S\_f} := \text{tB\_S\_f}^T$$

$$\text{B\_S\_f} = \begin{pmatrix} -1.95 & 0.356 \\ -1.306 & 0.409 \\ -0.653 & 0.44 \\ 0 & 0.45 \\ 0.653 & 0.44 \\ 1.306 & 0.409 \\ 1.95 & 0.356 \end{pmatrix} \text{cun}$$

Legend for plot:

$\dfrac{\text{B\_height}(X)}{\text{cun}}$ (red solid line)

$\dfrac{\text{B\_base}(X)}{\text{cun}}$ (blue dotted line)

$\dfrac{\text{B\_S\_bt}^{\langle 2 \rangle}}{\text{cun}}$ (×××)

$\dfrac{\text{B\_S\_f}^{\langle 2 \rangle}}{\text{cun}}$ (+++)

Horizontal axis: $\dfrac{X}{\text{cun}}, \dfrac{X}{\text{cun}}, \dfrac{\text{B\_S\_bt}^{\langle 1 \rangle}}{\text{cun}}, \dfrac{\text{B\_S\_f}^{\langle 1 \rangle}}{\text{cun}}$
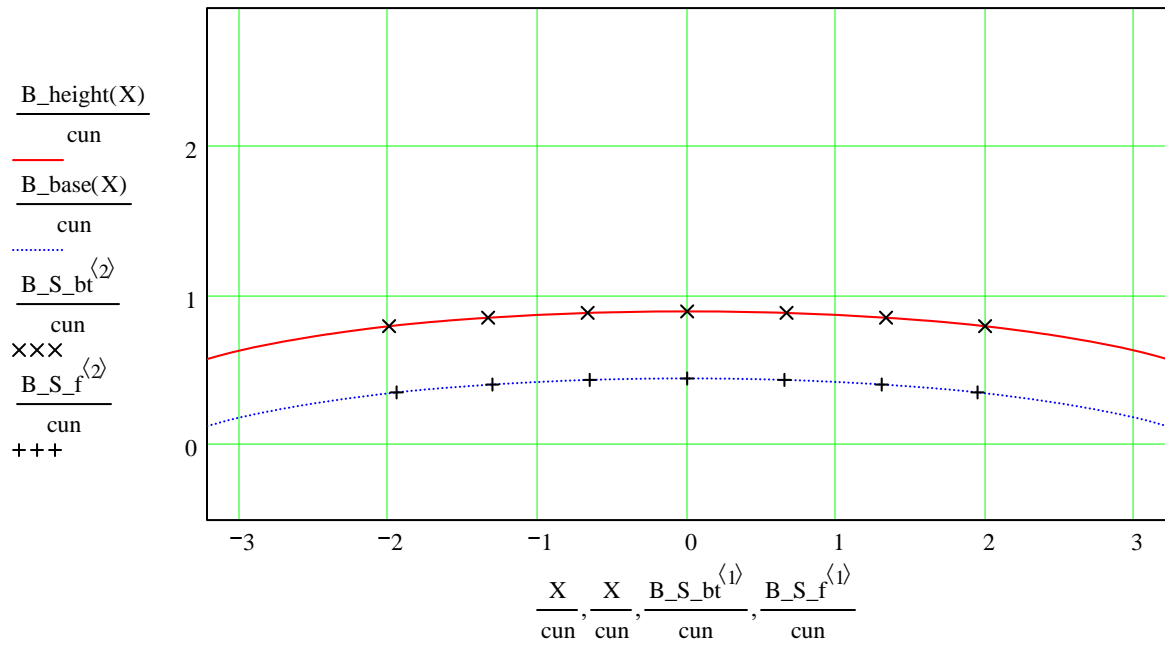
V.E.2.b.   Ensuring even distances between each Tuning Hole measured along the curvature at the base of the Bridge

Calculation of the Distance from the Axis to Each String Along the Face of the GuQin

$$\text{B\_swf}_1 := \int_{0\text{cun}}^{\text{B\_S\_f}_{1,1}} \sqrt{\left[1 + \left(\frac{d}{dX} \text{B\_base}(X) \right)^2 \right]}\, dX$$

$$\text{B\_swf}_2 := \int_{0\text{cun}}^{\text{B\_S\_f}_{2,1}} \sqrt{\left[1 + \left(\frac{d}{dX} \text{B\_base}(X) \right)^2 \right]}\, dX$$

$$B\_swf_3 := \int_{0cun}^{B\_S\_f_{3,1}} \sqrt{\left[1 + \left(\frac{d}{dX}B\_base(X)\right)^2\right]}\, dX$$

$$B\_swf_4 := \int_{0cun}^{B\_S\_f_{4,1}} \sqrt{\left[1 + \left(\frac{d}{dX}B\_base(X)\right)^2\right]}\, dX$$

$$B\_swf_5 := \int_{0cun}^{B\_S\_f_{5,1}} \sqrt{\left[1 + \left(\frac{d}{dX}B\_base(X)\right)^2\right]}\, dX$$

$$B\_swf_6 := \int_{0cun}^{B\_S\_f_{6,1}} \sqrt{\left[1 + \left(\frac{d}{dX}B\_base(X)\right)^2\right]}\, dX$$

$$B\_swf_7 := \int_{0cun}^{B\_S\_f_{7,1}} \sqrt{\left[1 + \left(\frac{d}{dX}B\_base(X)\right)^2\right]}\, dX$$

The distance from the axis to each string

$$B\_swf = \begin{pmatrix} -1.952617 \\ -1.3069031 \\ -0.6534555 \\ 0 \\ 0.6534555 \\ 1.3069031 \\ 1.952617 \end{pmatrix} cun$$

VI. Deriving the Curvature Functions for transverse sections of the GuQin

VI.A. Deriving the Curvature Functions from Shoulder to Nut
$C_n(x)$ where n=incremental distance from the Shoulder

Inserting uprise toward the tail

$$\begin{pmatrix} 25 & 0 \\ 26 & 0 \\ 27 & 0 \end{pmatrix}$$

$$\text{Uprise} := \begin{vmatrix} 28 & 0 \\ 29 & 0 \\ 29.648 & 0 \\ 39.64797372 & 0.022925775 \\ 49.64788487 & 0.065080186 \\ 59.64771251 & 0.123793003 \\ 69.64744423 & 0.197045233 \\ 79.64707262 & 0.283256765 \\ 89.64659341 & 0.381157299 \end{vmatrix} \cdot cm$$

$$\text{UpriseCurveConstant} := \text{lspline}\left(\text{Uprise}^{\langle 1 \rangle}, \text{Uprise}^{\langle 2 \rangle}\right)$$

$$\text{UC(X)} := \text{interp}\left(\text{UpriseCurveConstant}, \text{Uprise}^{\langle 1 \rangle}, \text{Uprise}^{\langle 2 \rangle}, X\right)$$

Derivation of Curvature Functions Cn(X), From Line 07 to Tail, in 1 inch increments

$$C0(X) := \text{Surface}\left(X, \text{Width}_7, \text{Height\_c}\right)$$

$$XY1 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7,12)} \cdot 1in + S\_XY \qquad X1 := XY1^{\langle 1 \rangle} \qquad Y1 := XY1^{\langle 2 \rangle}$$

$$S1 := \text{pspline}(X1, Y1) \qquad \text{Spline vector constant}$$

$$C1(X) := \text{interp}(S1, X1, Y1, X) \qquad$$ Curvature at 1 inch away from the shoulder, obtained by parabolic interpolation

$$XY2 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7,12)} \cdot 2in + S\_XY \qquad X2 := XY2^{\langle 1 \rangle} \qquad Y2 := XY2^{\langle 2 \rangle}$$

$$S2 := \text{pspline}(X2, Y2)$$

$$C2(X) := \text{interp}(S2, X2, Y2, X)$$

$$XY3 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7,12)} \cdot 3in + S\_XY \qquad X3 := XY3^{\langle 1 \rangle} \qquad Y3 := XY3^{\langle 2 \rangle}$$

$$S3 := \text{pspline}(X3, Y3)$$

$$C3(X) := \text{interp}(S3, X3, Y3, X)$$

$$XY4 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7,12)} \cdot 4in + S\_XY \qquad X4 := XY4^{\langle 1 \rangle} \qquad Y4 := XY4^{\langle 2 \rangle}$$

$$S4 := \text{pspline}(X4, Y4)$$

$$C4(X) := interp(S4, X4, Y4, X)$$

$$XY5 := \frac{(N\_XY - S\_XY)}{Distance(7, 12)} \cdot 5in + S\_XY \qquad X5 := XY5^{\langle 1 \rangle} \qquad Y5 := XY5^{\langle 2 \rangle}$$

$$S5 := pspline(X5, Y5)$$

$$C5(X) := interp(S5, X5, Y5, X)$$

$$XY6 := \frac{(N\_XY - S\_XY)}{Distance(7, 12)} \cdot 6in + S\_XY \qquad X6 := XY6^{\langle 1 \rangle} \qquad Y6 := XY6^{\langle 2 \rangle}$$

$$S6 := pspline(X6, Y6)$$

$$C6(X) := interp(S6, X6, Y6, X)$$

$$XY7 := \frac{(N\_XY - S\_XY)}{Distance(7, 12)} \cdot 7in + S\_XY \qquad X7 := XY7^{\langle 1 \rangle} \qquad Y7 := XY7^{\langle 2 \rangle}$$

$$S7 := pspline(X7, Y7)$$

$$C7(X) := interp(S7, X7, Y7, X)$$

$$XY8 := \frac{(N\_XY - S\_XY)}{Distance(7, 12)} \cdot 8in + S\_XY \qquad X8 := XY8^{\langle 1 \rangle} \qquad Y8 := XY8^{\langle 2 \rangle}$$

$$S8 := pspline(X8, Y8)$$

$$C8(X) := interp(S8, X8, Y8, X)$$

$$XY9 := \frac{(N\_XY - S\_XY)}{Distance(7, 12)} \cdot 9in + S\_XY \qquad X9 := XY9^{\langle 1 \rangle} \qquad Y9 := XY9^{\langle 2 \rangle}$$

$$S9 := pspline(X9, Y9)$$

$$C9(X) := interp(S9, X9, Y9, X)$$

$$XY10 := \frac{(N\_XY - S\_XY)}{Distance(7, 12)} \cdot 10in + S\_XY \qquad X10 := XY10^{\langle 1 \rangle} \qquad Y10 := XY10^{\langle 2 \rangle}$$

$$S10 := pspline(X10, Y10)$$

$$C10(X) := interp(S10, X10, Y10, X)$$

$$XY11 := \frac{(N\_XY - S\_XY)}{Distance(7, 12)} \cdot 11in + S\_XY \qquad X11 := XY11^{\langle 1 \rangle} \qquad Y11 := XY11^{\langle 2 \rangle}$$

$$S11 := pspline(X11, Y11)$$

$$C11(X) := \text{interp}(S11, X11, Y11, X)$$

$$XY12 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 12\text{in} + S\_XY \quad X12 := XY12^{\langle 1 \rangle} \quad Y12 := XY12^{\langle 2 \rangle}$$

$$S12 := \text{pspline}(X12, Y12)$$

$$C12(X) := \text{interp}(S12, X12, Y12, X) + UC(12\text{in})$$

$$XY13 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 13\text{in} + S\_XY \quad X13 := XY13^{\langle 1 \rangle} \quad Y13 := XY13^{\langle 2 \rangle}$$

$$S13 := \text{pspline}(X13, Y13)$$

$$C13(X) := \text{interp}(S13, X13, Y13, X) + UC(13\text{in})$$

$$XY14 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 14\text{in} + S\_XY \quad X14 := XY14^{\langle 1 \rangle} \quad Y14 := XY14^{\langle 2 \rangle}$$

$$S14 := \text{pspline}(X14, Y14)$$

$$C14(X) := \text{interp}(S14, X14, Y14, X) + UC(14\text{in})$$

$$XY15 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 15\text{in} + S\_XY \quad X15 := XY15^{\langle 1 \rangle} \quad Y15 := XY15^{\langle 2 \rangle}$$

$$S15 := \text{pspline}(X15, Y15)$$

$$C15(X) := \text{interp}(S15, X15, Y15, X) + UC(15\text{in})$$

$$XY16 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 16\text{in} + S\_XY \quad X16 := XY16^{\langle 1 \rangle} \quad Y16 := XY16^{\langle 2 \rangle}$$

$$S16 := \text{pspline}(X16, Y16)$$

$$C16(X) := \text{interp}(S16, X16, Y16, X) + UC(16\text{in})$$

$$XY17 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 17\text{in} + S\_XY \quad X17 := XY17^{\langle 1 \rangle} \quad Y17 := XY17^{\langle 2 \rangle}$$

$$S17 := \text{pspline}(X17, Y17)$$

$$C17(X) := \text{interp}(S17, X17, Y17, X) + UC(17\text{in})$$

$$XY18 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 18\text{in} + S\_XY \quad X18 := XY18^{\langle 1 \rangle} \quad Y18 := XY18^{\langle 2 \rangle}$$

$$S18 := \text{pspline}(X18, Y18)$$

$$C18(X) := \text{interp}(S18, X18, Y18, X) + UC(18\text{in})$$

$$XY19 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 19\text{in} + S\_XY \quad X19 := XY19^{\langle 1 \rangle} \quad Y19 := XY19^{\langle 2 \rangle}$$

$$S19 := \text{pspline}(X19, Y19)$$

$$C19(X) := \text{interp}(S19, X19, Y19, X) + UC(19\text{in})$$

$$XY20 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 20\text{in} + S\_XY \quad X20 := XY20^{\langle 1 \rangle} \quad Y20 := XY20^{\langle 2 \rangle}$$

$$S20 := \text{pspline}(X20, Y20)$$

$$C20(X) := \text{interp}(S20, X20, Y20, X) + UC(20\text{in})$$

$$XY21 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 21\text{in} + S\_XY \quad X21 := XY21^{\langle 1 \rangle} \quad Y21 := XY21^{\langle 2 \rangle}$$

$$S21 := \text{pspline}(X21, Y21)$$

$$C21(X) := \text{interp}(S21, X21, Y21, X) + UC(21\text{in})$$

$$XY22 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 22\text{in} + S\_XY \quad X22 := XY22^{\langle 1 \rangle} \quad Y22 := XY22^{\langle 2 \rangle}$$

$$S22 := \text{pspline}(X22, Y22)$$

$$C22(X) := \text{interp}(S22, X22, Y22, X) + UC(22\text{in})$$

$$XY23 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 23\text{in} + S\_XY \quad X23 := XY23^{\langle 1 \rangle} \quad Y23 := XY23^{\langle 2 \rangle}$$

$$S23 := \text{pspline}(X23, Y23)$$

$$C23(X) := \text{interp}(S23, X23, Y23, X) + UC(23\text{in})$$

$$XY24 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 24\text{in} + S\_XY \quad X24 := XY24^{\langle 1 \rangle} \quad Y24 := XY24^{\langle 2 \rangle}$$

$$S24 := \text{pspline}(X24, Y24)$$

$$C24(X) := \text{interp}(S24, X24, Y24, X) + UC(24\text{in})$$

$$XY25 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 25\text{in} + S\_XY \quad X25 := XY25^{\langle 1 \rangle} \quad Y25 := XY25^{\langle 2 \rangle}$$

$$S25 := \text{pspline}(X25, Y25)$$

$$C25(X) := \text{interp}(S25, X25, Y25, X) + UC(25\text{in})$$

$$XY26 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 26\text{in} + S\_XY \quad X26 := XY26^{\langle 1 \rangle} \quad Y26 := XY26^{\langle 2 \rangle}$$

$$S26 := \text{pspline}(X26, Y26)$$

$$C26(X) := \text{interp}(S26, X26, Y26, X) + UC(26\text{in})$$

$$XY27 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 27\text{in} + S\_XY \quad X27 := XY27^{\langle 1 \rangle} \quad Y27 := XY27^{\langle 2 \rangle}$$

$$S27 := \text{pspline}(X27, Y27)$$

$$C27(X) := \text{interp}(S27, X27, Y27, X) + UC(27\text{in})$$

$$XY28 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 28\text{in} + S\_XY \quad X28 := XY28^{\langle 1 \rangle} \quad Y28 := XY28^{\langle 2 \rangle}$$

$$S28 := \text{pspline}(X28, Y28)$$

$$C28(X) := \text{interp}(S28, X28, Y28, X) + UC(28\text{in})$$

$$XY29 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 29\text{in} + S\_XY \quad X29 := XY29^{\langle 1 \rangle} \quad Y29 := XY29^{\langle 2 \rangle}$$

$$S29 := \text{pspline}(X29, Y29)$$

$$C29(X) := \text{interp}(S29, X29, Y29, X) + UC(29\text{in})$$

$$XY30 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 30\text{in} + S\_XY \quad X30 := XY30^{\langle 1 \rangle} \quad Y30 := XY30^{\langle 2 \rangle}$$

$$S30 := \text{pspline}(X30, Y30)$$

$$C30(X) := \text{interp}(S30, X30, Y30, X) + UC(30\text{in})$$

$$XY31 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 31\text{in} + S\_XY \quad X31 := XY31^{\langle 1 \rangle} \quad Y31 := XY31^{\langle 2 \rangle}$$

$$S31 := \text{pspline}(X31, Y31)$$

$$C31(X) := \text{interp}(S31, X31, Y31, X) + UC(31\text{in})$$

$$XY32 := \frac{(N\_XY - S\_XY)}{\text{Distance}(7, 12)} \cdot 32\text{in} + S\_XY \quad X32 := XY32^{\langle 1 \rangle} \quad Y32 := XY32^{\langle 2 \rangle}$$

$$S32 := \text{pspline}(X32, Y32)$$

$$C32(X) := \text{interp}(S32, X32, Y32, X) + UC(32\text{in})$$

$$XY33 := \frac{(N\_XY - S\_XY)}{Distance(7,12)} \cdot 33in + S\_XY \quad X33 := XY33^{\langle 1 \rangle} \quad Y33 := XY33^{\langle 2 \rangle}$$

$$S33 := pspline(X33, Y33)$$

$$C33(X) := interp(S33, X33, Y33, X) + UC(33in)$$

$$XY34 := \frac{(N\_XY - S\_XY)}{Distance(7,12)} \cdot 34in + S\_XY \quad X34 := XY34^{\langle 1 \rangle} \quad Y34 := XY34^{\langle 2 \rangle}$$

$$S34 := pspline(X34, Y34)$$

$$C34(X) := interp(S34, X34, Y34, X) + UC(34in)$$

$$XY35 := \frac{(N\_XY - S\_XY)}{Distance(7,12)} \cdot 35in + S\_XY \quad X35 := XY35^{\langle 1 \rangle} \quad Y35 := XY35^{\langle 2 \rangle}$$

$$S35 := pspline(X35, Y35)$$

$$C35(X) := interp(S35, X35, Y35, X) + UC(35 \cdot in)$$

$$ORIGIN := 0$$

$$p := \begin{pmatrix} -4 \\ -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

$TD0 := C0(p \cdot in)$	$TD1 := C1(p\,in)$	$TD2 := C2(p\,in)$	$TD3 := C3(p\,in)$
$TD4 := C4(p\,in)$	$TD5 := C5(p\,in)$	$TD6 := C6(p\,in)$	$TD7 := C7(p\,in)$
$TD8 := C8(p\,in)$	$TD9 := C9(p\,in)$	$TD10 := C10(p\,in)$	$TD11 := C11(p\,in)$
$TD12 := C12(p\,in)$	$TD13 := C13(p\,in)$	$TD14 := C14(p\,in)$	$TD15 := C15(p\,in)$
$TD16 := C16(p\,in)$	$TD17 := C17(p\,in)$	$TD18 := C18(p\,in)$	$TD19 := C19(p\,in)$
$TD20 := C20(p\,in)$	$TD21 := C21(p\,in)$	$TD22 := C22(p\,in)$	$TD23 := C23(p\,in)$
$TD24 := C24(p\,in)$	$TD25 := C25(p\,in)$	$TD26 := C26(p\,in)$	$TD27 := C27(p\,in)$
$TD28 := C28(p \cdot in)$	$TD29 := C29(p\,in)$	$TD30 := C30(p\,in)$	$TD31 := C31(p\,in)$

$$TD32 := C32(p \text{ in}) \quad TD33 := C33(p \text{ in}) \quad TD34 := C34(p \text{ in}) \quad TD35 := C35(p \cdot in)$$

$$TD^{\langle 1 \rangle} := TD1 \qquad\qquad TD^{\langle 2 \rangle} := TD2$$
$$TD^{\langle 3 \rangle} := TD3 \qquad\qquad TD^{\langle 4 \rangle} := TD4$$
$$TD^{\langle 5 \rangle} := TD5 \qquad\qquad TD^{\langle 6 \rangle} := TD6$$
$$TD^{\langle 7 \rangle} := TD7 \qquad\qquad TD^{\langle 8 \rangle} := TD8$$
$$TD^{\langle 9 \rangle} := TD9 \qquad\qquad TD^{\langle 10 \rangle} := TD10$$
$$TD^{\langle 11 \rangle} := TD11 \qquad\qquad TD^{\langle 12 \rangle} := TD12$$
$$TD^{\langle 13 \rangle} := TD13 \qquad\qquad TD^{\langle 14 \rangle} := TD14$$
$$TD^{\langle 15 \rangle} := TD15 \qquad\qquad TD^{\langle 16 \rangle} := TD16$$
$$TD^{\langle 17 \rangle} := TD17 \qquad\qquad TD^{\langle 18 \rangle} := TD18$$
$$TD^{\langle 19 \rangle} := TD19 \qquad\qquad TD^{\langle 20 \rangle} := TD20$$
$$TD^{\langle 21 \rangle} := TD21 \qquad\qquad TD^{\langle 22 \rangle} := TD22$$
$$TD^{\langle 23 \rangle} := TD23 \qquad\qquad TD^{\langle 24 \rangle} := TD24$$
$$TD^{\langle 25 \rangle} := TD25 \qquad\qquad TD^{\langle 26 \rangle} := TD26$$
$$TD^{\langle 27 \rangle} := TD27 \qquad\qquad TD^{\langle 28 \rangle} := TD28$$
$$TD^{\langle 29 \rangle} := TD29 \qquad\qquad TD^{\langle 30 \rangle} := TD30$$
$$TD^{\langle 31 \rangle} := TD31 \qquad\qquad TD^{\langle 32 \rangle} := TD32$$
$$TD^{\langle 33 \rangle} := TD33 \qquad\qquad TD^{\langle 34 \rangle} := TD34$$
$$TD^{\langle 35 \rangle} := TD35$$
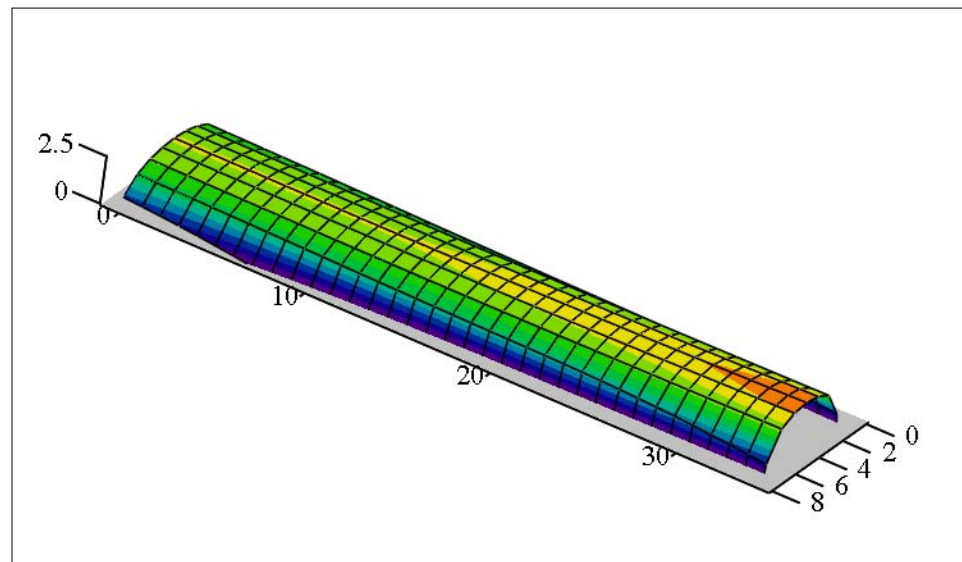
TD

VI.B. Deriving the Curvature Functions from Shoulder to the First Hui
$rCn(x)$ where n=incremental distance from the Shoulder

$$\text{ORIGIN} := 1$$

Derivation of Curvature Functions Crn(X), From Line 07 to First Hui, in 1 inch increments

$$\text{D\_BtoHui1} := \frac{\text{Distance}(4, 12)}{8}$$

$$\text{D\_toHui1} := \text{Dist}_4 + \text{D\_BtoHui1}$$

$$\text{D\_toHui1} = 7.2 \, \text{cun}$$

$$\text{Dist}_7 - \text{D\_toHui1} = 3.3 \, \text{in}$$

$$\text{rXY1} := \frac{(\text{N\_XY} - \text{S\_XY})}{\text{Distance}(7, 12)} \cdot -1\text{in} + \text{S\_XY} \qquad \text{rX1} := \text{rXY1}^{\langle 1 \rangle} \qquad \text{rY1} := \text{rXY1}^{\langle 2 \rangle}$$

$$\text{rS1} := \text{pspline}(\text{rX1}, \text{rY1})$$

$$\text{rC1}(X) := \text{interp}(\text{rS1}, \text{rX1}, \text{rY1}, X)$$

$$\text{rXY2} := \frac{(\text{N\_XY} - \text{S\_XY})}{\text{Distance}(7, 12)} \cdot -2\text{in} + \text{S\_XY} \qquad \text{rX2} := \text{rXY2}^{\langle 1 \rangle} \qquad \text{rY2} := \text{rXY2}^{\langle 2 \rangle}$$

$$\text{rS2} := \text{pspline}(\text{rX2}, \text{rY2})$$

$$\text{rC2}(X) := \text{interp}(\text{rS2}, \text{rX2}, \text{rY2}, X)$$

$$\text{rXY3} := \frac{(\text{N\_XY} - \text{S\_XY})}{\text{Distance}(7, 12)} \cdot -3\text{in} + \text{S\_XY} \qquad \text{rX3} := \text{rXY3}^{\langle 1 \rangle} \qquad \text{rY3} := \text{rXY3}^{\langle 2 \rangle}$$

$$\text{rS3} := \text{pspline}(\text{rX3}, \text{rY3})$$

$$\text{rC3}(X) := \text{interp}(\text{rS3}, \text{rX3}, \text{rY3}, X)$$

VI.C. Deriving the Curvature Function at the First Hui
$CHui1(x)$

$$\text{XYHui1} := \frac{(\text{N\_XY} - \text{S\_XY})}{\text{Distance}(7, 12)} \cdot -3.3\text{in} + \text{S\_XY} \qquad \text{XHui1} := \text{XYHui1}^{\langle 1 \rangle}$$

$$\text{YHui1} := \text{XYHui1}^{\langle 2 \rangle}$$

$$\text{SHui1} := \text{pspline}(\text{XHui1}, \text{YHui1})$$

$$\text{CHui1}(X) := \text{interp}(\text{SHui1}, \text{XHui1}, \text{YHui1}, X)$$

VI.D. Deriving the Curvature Functions from Bridge to the First Hui
$CBHn(x)$ where n=incremental distance from the Bridge

Derivation of Curvature Functions CBHn(X), From Bridge to First Hui, in 1 inch increments

$$\text{interp\_XYBH1} := \frac{(S\_XY - N\_XY)}{\text{Distance}(7,12)} \cdot (\text{Distance}(4,12) - 1\text{in}) + N\_XY$$

$$\text{XBH1} := \text{interp\_XYBH1}^{\langle 1 \rangle} \qquad \text{YBH1} := \text{interp\_XYBH1}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(D\_BtoHui1 - 1\text{in})}{D\_BtoHui1}$$

$$\text{SBH1} := \text{pspline}(\text{XBH1}, \text{YBH1})$$

$$\text{CBH1}(X) := \text{interp}(\text{SBH1}, \text{XBH1}, \text{YBH1}, X)$$

$$\text{interp\_XYBH2} := \frac{(S\_XY - N\_XY)}{\text{Distance}(7,12)} \cdot (\text{Distance}(4,12) - 2\text{in}) + N\_XY$$

$$\text{XBH2} := \text{interp\_XYBH2}^{\langle 1 \rangle} \qquad \text{YBH2} := \text{interp\_XYBH2}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(D\_BtoHui1 - 2\text{in})}{D\_BtoHui1}$$

$$\text{SBH2} := \text{pspline}(\text{XBH2}, \text{YBH2})$$

$$\text{CBH2}(X) := \text{interp}(\text{SBH2}, \text{XBH2}, \text{YBH2}, X)$$

$$\text{interp\_XYBH3} := \frac{(S\_XY - N\_XY)}{\text{Distance}(7,12)} \cdot (\text{Distance}(4,12) - 3\text{in}) + N\_XY$$

$$\text{XBH3} := \text{interp\_XYBH3}^{\langle 1 \rangle} \qquad \text{YBH3} := \text{interp\_XYBH3}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(D\_BtoHui1 - 3\text{in})}{D\_BtoHui1}$$

$$\text{SBH3} := \text{pspline}(\text{XBH3}, \text{YBH3})$$

$$\text{CBH3}(X) := \text{interp}(\text{SBH3}, \text{XBH3}, \text{YBH3}, X)$$

$$\text{interp\_XYBH4} := \frac{(S\_XY - N\_XY)}{\text{Distance}(7,12)} \cdot (\text{Distance}(4,12) - 4\text{in}) + N\_XY$$

$$\text{XBH4} := \text{interp\_XYBH4}^{\langle 1 \rangle} \qquad \text{YBH4} := \text{interp\_XYBH4}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(D\_BtoHui1 - 4\text{in})}{D\_BtoHui1}$$

$$\text{SBH4} := \text{pspline}(\text{XBH4}, \text{YBH4})$$

$$\text{CBH4}(X) := \text{interp}(\text{SBH4}, \text{XBH4}, \text{YBH4}, X)$$

$$\text{interp\_XYBH5} := \frac{(S\_XY - N\_XY)}{\text{Distance}(7,12)} \cdot (\text{Distance}(4,12) - 5\text{in}) + N\_XY$$

$$\text{XBH5} := \text{interp\_XYBH5}^{\langle 1 \rangle} \qquad \text{YBH5} := \text{interp\_XYBH5}^{\langle 2 \rangle} - 0.2\text{cun} \cdot \frac{(D\_BtoHui1 - 5\text{in})}{D\_BtoHui1}$$

$$\text{SBH5} := \text{pspline}(\text{XBH5}, \text{YBH5})$$

$$\text{CBH5}(X) := \text{interp}(\text{SBH5}, \text{XBH5}, \text{YBH5}, X)$$

VI.E. **Including the Curvature Function at the base of the Bridge**
B_base(x)

(See section V.D.)

VII. Calculating drilling depths of $^{1}/_{16}$ inch increments

Solving for Drilling Depth Locations

$$d := \frac{1}{16} in \qquad Wood\_width := 0.65 cun \qquad inc := 1 .. 10$$

$$d_{inc} := d \cdot inc \qquad x_{inc} := 1.5 cun$$

VII.A. Specifying positions of each drilling depth from Shoulder to Nut
Depth_Shoulder_to_Nut

From Shoulder to Nut, in 1 inch intervals

$$Drill := \frac{1}{32} in \qquad WTh := 2.4 cm \qquad incr := 1 .. 36$$

$$Dr_{incr} := Drill \cdot incr \qquad ID := 2 \cdot cm$$

$$ID := root(WTh - Dr_1 - C35(ID), ID) \qquad ID = 2.013 \ cm$$
$$ID := root(WTh - Dr_2 - C35(ID), ID) \qquad ID = 2.902 \ cm$$
$$ID := root(WTh - Dr_3 - C35(ID), ID) \qquad ID = 3.554 \ cm$$
$$ID := root(WTh - Dr_4 - C35(ID), ID) \qquad ID = 4.073 \ cm$$
$$ID := root(WTh - Dr_5 - C35(ID), ID) \qquad ID = 4.52 \ cm$$
$$ID := root(WTh - Dr_6 - C35(ID), ID) \qquad ID = 4.913 \ cm$$
$$ID := root(WTh - Dr_7 - C35(ID), ID) \qquad ID = 5.255 \ cm$$
$$ID := root(WTh - Dr_8 - C35(ID), ID) \qquad ID = 5.547 \ cm$$
$$ID := root(WTh - Dr_9 - C35(ID), ID) \qquad ID = 5.808 \ cm$$
$$ID := root(WTh - Dr_{10} - C35(ID), ID) \qquad ID = 6.061 \ cm$$
$$ID := root(WTh - Dr_{11} - C35(ID), ID) \qquad ID = 6.313 \ cm$$
$$ID := root(WTh - Dr_{12} - C35(ID), ID) \qquad ID = 6.541 \ cm$$
$$ID := root(WTh - Dr_{13} - C35(ID), ID) \qquad ID = 6.717 \ cm$$
$$ID := root(WTh - Dr_{14} - C35(ID), ID) \qquad ID = 6.854 \ cm$$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{15} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 6.965 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{16} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.061 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{17} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.146 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{18} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.223 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{19} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.294 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{20} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.36 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{21} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.422 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{22} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.481 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{24} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.59 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{26} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.691 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{30} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 7.873 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{34} - \text{C35(ID)}, \text{ID}\right)$ $\text{ID} = 8.036 \text{ cm}$

$\text{ID} := 1.624 \text{cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{1} - \text{C34(ID)}, \text{ID}\right)$ $\text{ID} = 1.624 \text{ cm}$

$\text{ID} := 1.12 \text{cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{1} - \text{C33(ID)}, \text{ID}\right)$ $\text{ID} = 1.106 \text{ cm}$

$\text{ID} := 2.1 \text{cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{2} - \text{C32(ID)}, \text{ID}\right)$ $\text{ID} = 2.117 \text{ cm}$

$\text{ID} := 1.3 \text{cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{2} - \text{C30(ID)}, \text{ID}\right)$ $\text{ID} = 1.349 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{3} - \text{C30(ID)}, \text{ID}\right)$ $\text{ID} = 2.598 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{4} - \text{C30(ID)}, \text{ID}\right)$ $\text{ID} = 3.432 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{5} - \text{C30(ID)}, \text{ID}\right)$ $\text{ID} = 4.061 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{6} - \text{C30(ID)}, \text{ID}\right)$ $\text{ID} = 4.572 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{7} - \text{C30(ID)}, \text{ID}\right)$ $\text{ID} = 5.014 \text{ cm}$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{8} - \text{C30(ID)}, \text{ID}\right)$ $\text{ID} = 5.405 \text{ cm}$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_9 - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 5.746\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{10} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 6.038\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{11} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 6.3\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{12} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 6.554\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{13} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 6.807\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{14} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.033\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{15} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.205\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{16} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.34\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{17} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.449\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{18} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.543\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{19} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.627\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{20} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.704\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{21} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.774\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{22} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.84\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{24} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 7.96\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{26} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 8.07\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{30} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 8.265\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_{34} - \mathrm{C30(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 8.436\,\mathrm{cm}$$

$$\mathrm{ID} := 0.7\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_2 - \mathrm{C29(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 0.66\,\mathrm{cm}$$

$$\mathrm{ID} := 1.7\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_3 - \mathrm{C27(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 1.73\,\mathrm{cm}$$

$$\mathrm{ID} := 1.3\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_3 - \mathrm{C26(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 1.327\,\mathrm{cm}$$

$$\mathrm{ID} := 0.7\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_3 - \mathrm{C25(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 0.677\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_4 - \mathrm{C25(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 2.418\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_5 - \mathrm{C25(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 3.399\,\mathrm{cm}$$

$$\mathrm{ID} := \mathrm{root}\left(\mathrm{WTh} - \mathrm{Dr}_6 - \mathrm{C25(ID)}, \mathrm{ID}\right) \qquad \mathrm{ID} = 4.125\,\mathrm{cm}$$

$$ID := \text{root}\left(WTh - Dr_7 - C25(ID), ID\right) \qquad ID = 4.699 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_8 - C25(ID), ID\right) \qquad ID = 5.186 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_9 - C25(ID), ID\right) \qquad ID = 5.611 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{10} - C25(ID), ID\right) \qquad ID = 5.988 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{11} - C25(ID), ID\right) \qquad ID = 6.315 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{12} - C25(ID), ID\right) \qquad ID = 6.597 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{13} - C25(ID), ID\right) \qquad ID = 6.856 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{14} - C25(ID), ID\right) \qquad ID = 7.111 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{15} - C25(ID), ID\right) \qquad ID = 7.366 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{16} - C25(ID), ID\right) \qquad ID = 7.572 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{17} - C25(ID), ID\right) \qquad ID = 7.729 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{18} - C25(ID), ID\right) \qquad ID = 7.853 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{19} - C25(ID), ID\right) \qquad ID = 7.957 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{20} - C25(ID), ID\right) \qquad ID = 8.048 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{21} - C25(ID), ID\right) \qquad ID = 8.129 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{22} - C25(ID), ID\right) \qquad ID = 8.203 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{23} - C25(ID), ID\right) \qquad ID = 8.272 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{24} - C25(ID), ID\right) \qquad ID = 8.337 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{25} - C25(ID), ID\right) \qquad ID = 8.398 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{26} - C25(ID), ID\right) \qquad ID = 8.456 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{30} - C25(ID), ID\right) \qquad ID = 8.664 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{34} - C25(ID), ID\right) \qquad ID = 8.845 \text{ cm}$$

$$ID := 1.25 \text{cm}$$

$$ID := \text{root}\left(WTh - Dr_4 - C21(ID), ID\right) \qquad ID = 1.259 \text{ cm}$$

$$ID := 0.7 \text{cm}$$

$$ID := \text{root}\left(WTh - Dr_4 - C20(ID), ID\right) \qquad ID = 0.709 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_5 - C20(ID), ID\right) \qquad ID = 2.561 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_6 - C20(ID), ID\right) \qquad ID = 3.556 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_7 - C20(ID), ID\right) \qquad ID = 4.34 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_8 - C20(ID), ID\right) \qquad ID = 4.949 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_9 - C20(ID), ID\right) \qquad ID = 5.467 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{10} - C20(ID), ID\right) \qquad ID = 5.913 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{11} - C20(ID), ID\right) \qquad ID = 6.31 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{12} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 6.659 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{13} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 6.96 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{14} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 7.228 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{15} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 7.487 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{16} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 7.744 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{17} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 7.975 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{18} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.149 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{19} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.284 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{20} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.394 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{21} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.489 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{22} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.574 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{23} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.651 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{24} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.722 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{25} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.788 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{26} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.851 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{27} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.91 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{28} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 8.966 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{29} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 9.02 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{30} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 9.072 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{34} - \text{C20(ID)}, \text{ID}\right) \qquad ID = 9.262 \text{ cm}$$

$$ID := 1.69 \text{cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{5} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 1.694 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{6} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 3.079 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{7} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 4.004 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{8} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 4.775 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{9} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 5.38 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{10} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 5.899 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{11} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 6.348 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{12} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 6.749 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{13} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 7.104 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{14} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 7.411 \text{ cm}$$

$$ID := \text{root}\left(\text{WTh} - \text{Dr}_{15} - \text{C15(ID)}, \text{ID}\right) \qquad ID = 7.683 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{16} - C15(ID), ID\right) \qquad ID = 7.945 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{17} - C15(ID), ID\right) \qquad ID = 8.207 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{18} - C15(ID), ID\right) \qquad ID = 8.44 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{19} - C15(ID), ID\right) \qquad ID = 8.616 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{20} - C15(ID), ID\right) \qquad ID = 8.753 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{21} - C15(ID), ID\right) \qquad ID = 8.863 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{22} - C15(ID), ID\right) \qquad ID = 8.959 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{23} - C15(ID), ID\right) \qquad ID = 9.045 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{24} - C15(ID), ID\right) \qquad ID = 9.122 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{25} - C15(ID), ID\right) \qquad ID = 9.194 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{26} - C15(ID), ID\right) \qquad ID = 9.261 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{27} - C15(ID), ID\right) \qquad ID = 9.323 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{28} - C15(ID), ID\right) \qquad ID = 9.383 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{29} - C15(ID), ID\right) \qquad ID = 9.44 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{30} - C15(ID), ID\right) \qquad ID = 9.494 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{31} - C15(ID), ID\right) \qquad ID = 9.546 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{32} - C15(ID), ID\right) \qquad ID = 9.597 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{33} - C15(ID), ID\right) \qquad ID = 9.645 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{34} - C15(ID), ID\right) \qquad ID = 9.692 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{35} - C15(ID), ID\right) \qquad ID = 9.738 \text{ cm}$$

$$ID := 1.2 \text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{5} - C10(ID), ID\right) \qquad ID = 1.2 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{6} - C10(ID), ID\right) \qquad ID = 3.009 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{7} - C10(ID), ID\right) \qquad ID = 4.01 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{8} - C10(ID), ID\right) \qquad ID = 4.823 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{9} - C10(ID), ID\right) \qquad ID = 5.5 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{10} - C10(ID), ID\right) \qquad ID = 6.058 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{11} - C10(ID), ID\right) \qquad ID = 6.544 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{12} - C10(ID), ID\right) \qquad ID = 6.971 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{13} - C10(ID), ID\right) \qquad ID = 7.357 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{14} - C10(ID), ID\right) \qquad ID = 7.696 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{15} - C10(ID), ID\right) \qquad ID = 7.989 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{16} - C10(ID), ID\right) \qquad ID = 8.257 \text{ cm}$$

$$ID := \text{root}\left(WTh - Dr_{17} - C10(ID), ID\right) \qquad ID = 8.519\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{18} - C10(ID), ID\right) \qquad ID = 8.781\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{19} - C10(ID), ID\right) \qquad ID = 8.992\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{20} - C10(ID), ID\right) \qquad ID = 9.15\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{21} - C10(ID), ID\right) \qquad ID = 9.274\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{22} - C10(ID), ID\right) \qquad ID = 9.379\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{23} - C10(ID), ID\right) \qquad ID = 9.47\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{24} - C10(ID), ID\right) \qquad ID = 9.552\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{25} - C10(ID), ID\right) \qquad ID = 9.627\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{26} - C10(ID), ID\right) \qquad ID = 9.697\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{27} - C10(ID), ID\right) \qquad ID = 9.762\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{28} - C10(ID), ID\right) \qquad ID = 9.824\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{29} - C10(ID), ID\right) \qquad ID = 9.883\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{30} - C10(ID), ID\right) \qquad ID = 9.939\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{31} - C10(ID), ID\right) \qquad ID = 9.993\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{32} - C10(ID), ID\right) \qquad ID = 10.044\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{33} - C10(ID), ID\right) \qquad ID = 10.094\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{34} - C10(ID), ID\right) \qquad ID = 10.142\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{35} - C10(ID), ID\right) \qquad ID = 10.189\,\text{cm}$$

$$ID := 1.258\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{5} - C5(ID), ID\right) \qquad ID = 1.257\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{6} - C5(ID), ID\right) \qquad ID = 3.156\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{7} - C5(ID), ID\right) \qquad ID = 4.209\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{8} - C5(ID), ID\right) \qquad ID = 5.035\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{9} - C5(ID), ID\right) \qquad ID = 5.73\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{10} - C5(ID), ID\right) \qquad ID = 6.318\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{11} - C5(ID), ID\right) \qquad ID = 6.826\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{12} - C5(ID), ID\right) \qquad ID = 7.273\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{13} - C5(ID), ID\right) \qquad ID = 7.675\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{14} - C5(ID), ID\right) \qquad ID = 8.035\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{15} - C5(ID), ID\right) \qquad ID = 8.348\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{16} - C5(ID), ID\right) \qquad ID = 8.626\,\text{cm}$$

$$ID := \text{root}\left(WTh - Dr_{17} - C5(ID), ID\right) \qquad ID = 8.893\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{18} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 9.158\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{19} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 9.395\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{20} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 9.57\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{21} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 9.706\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{22} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 9.816\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{23} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 9.912\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{24} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 9.998\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{25} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.075\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{26} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.147\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{27} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.215\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{28} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.278\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{29} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.338\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{30} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.396\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{31} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.45\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{32} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.503\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{33} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.554\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{34} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.603\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{35} - \text{C5(ID)}, \text{ID}\right) \qquad \text{ID} = 10.651\,\text{cm}$$

$$\text{ID} := 1.313\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{5} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 1.312\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{6} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 3.292\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{7} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 4.407\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{8} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 5.266\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{9} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 5.968\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{10} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 6.573\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{11} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 7.102\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{12} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 7.569\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{13} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 7.986\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{14} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 8.36\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{15} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 8.697\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{16} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 9.001\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{17} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 9.276\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{18} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 9.523\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{19} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 9.746\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{20} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 9.945 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{21} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.122 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{22} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.279 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{23} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.415 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{24} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.533 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{25} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.632 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{26} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.713 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{27} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.777 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{28} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.823 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{29} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.852 \, \text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{30} - \text{C0(ID)}, \text{ID}\right) \qquad \text{ID} = 10.865 \, \text{cm}$$

$$\text{AddedUC} := \begin{pmatrix} 0\text{in} & \text{C0(0·in)} \\ 5\text{in} & \text{C5(0·in)} \\ 10\text{in} & \text{C10(0in)} \\ 15\text{in} & \text{C15(0in)} \\ 20\text{in} & \text{C20(0in)} \\ 25\text{in} & \text{C25(0in)} \\ 30\text{in} & \text{C30(0in)} \\ 35\text{in} & \text{C35(0in)} \end{pmatrix}$$

$$\text{AddedUCConstant} := \text{lspline}\left(\text{AddedUC}^{\langle 1 \rangle}, \text{AddedUC}^{\langle 2 \rangle}\right)$$

$$\text{AddedUC(X)} := \text{interp}\left(\text{AddedUCConstant}, \text{AddedUC}^{\langle 1 \rangle}, \text{AddedUC}^{\langle 2 \rangle}, X\right)$$

$$\text{UCD} := 32\text{in}$$

$$\text{UCD} := \text{root}\left(\text{AddedUC(UCD)} + \text{Dr}_1 - \text{WTh}, \text{UCD}\right) \qquad \text{UCD} = 32.15 \, \text{in}$$

$$\text{UCD} := \text{root}\left(\text{AddedUC(UCD)} + \text{Dr}_2 - \text{WTh}, \text{UCD}\right) \qquad \text{UCD} = 28.713 \, \text{in}$$

$$\text{UCD} := \text{root}\left(\text{AddedUC(UCD)} + \text{Dr}_3 - \text{WTh}, \text{UCD}\right) \qquad \text{UCD} = 24.672 \, \text{in}$$

$$\text{UCD} := \text{root}\left(\text{AddedUC(UCD)} + \text{Dr}_4 - \text{WTh}, \text{UCD}\right) \qquad \text{UCD} = 19.53 \, \text{in}$$

VII.B. Specifying positions of each drilling depth at First Hui Depth_Hui1

At First Hui

$$ID := 1.349 \text{cm}$$

$ID := \text{root}\left(WTh - Dr_5 - CHui1(ID), ID\right)$ $\qquad ID = 1.349 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_6 - CHui1(ID), ID\right)$ $\qquad ID = 3.376 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_7 - CHui1(ID), ID\right)$ $\qquad ID = 4.536 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_8 - CHui1(ID), ID\right)$ $\qquad ID = 5.419 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_9 - CHui1(ID), ID\right)$ $\qquad ID = 6.127 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{10} - CHui1(ID), ID\right)$ $\qquad ID = 6.742 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{11} - CHui1(ID), ID\right)$ $\qquad ID = 7.281 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{12} - CHui1(ID), ID\right)$ $\qquad ID = 7.759 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{13} - CHui1(ID), ID\right)$ $\qquad ID = 8.186 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{14} - CHui1(ID), ID\right)$ $\qquad ID = 8.577 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{15} - CHui1(ID), ID\right)$ $\qquad ID = 8.925 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{16} - CHui1(ID), ID\right)$ $\qquad ID = 9.228 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{17} - CHui1(ID), ID\right)$ $\qquad ID = 9.504 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{18} - CHui1(ID), ID\right)$ $\qquad ID = 9.773 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{19} - CHui1(ID), ID\right)$ $\qquad ID = 10.042 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{20} - CHui1(ID), ID\right)$ $\qquad ID = 10.255 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{21} - CHui1(ID), ID\right)$ $\qquad ID = 10.414 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{22} - CHui1(ID), ID\right)$ $\qquad ID = 10.538 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{23} - CHui1(ID), ID\right)$ $\qquad ID = 10.643 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{24} - CHui1(ID), ID\right)$ $\qquad ID = 10.735 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{25} - CHui1(ID), ID\right)$ $\qquad ID = 10.818 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{26} - CHui1(ID), ID\right)$ $\qquad ID = 10.894 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{27} - CHui1(ID), ID\right)$ $\qquad ID = 10.965 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{28} - CHui1(ID), ID\right)$ $\qquad ID = 11.031 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{29} - CHui1(ID), ID\right)$ $\qquad ID = 11.094 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{30} - CHui1(ID), ID\right)$ $\qquad ID = 11.153 \text{ cm}$

$ID := \text{root}\left(WTh - Dr_{34} - CHui1(ID), ID\right)$ $\qquad ID = 11.368 \text{ cm}$

VII.C. Specifying positions of each drilling depth from Bridge to First Hui
Depth_Bridge_to_Hui1

From Bridge to First Hui, in 1 inch intervals

5 in from bridge $\quad\quad \text{ID} := 2.149\,\text{cm}$

$\text{Dist}_7 - \text{Dist}_4 - 5\text{in} = 3.8\,\text{in}$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_6 - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 2.153\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_7 - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 3.746\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_8 - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 4.835\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_9 - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 5.663\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{10} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 6.338\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{11} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 6.931\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{12} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 7.453\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{13} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 7.916\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{14} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 8.333\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{15} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 8.715\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{16} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 9.052\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{17} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 9.346\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{18} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 9.619\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{19} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 9.892\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{20} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 10.147\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{21} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 10.347\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{22} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 10.495\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{23} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 10.613\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{24} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 10.715\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{25} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 10.804\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{26} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 10.885\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{27} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 10.96\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{28} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 11.03\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{29} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 11.095\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{30} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 11.157\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{34} - \text{CBH5(ID)}, \text{ID}\right) \quad\quad \text{ID} = 11.378\,\text{cm}$$

4 in from bridge $ID := 0.73\text{cm}$

$\text{Dist}_7 - \text{Dist}_4 - 4\text{in} = 4.8\,\text{in}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_7 - \text{CBH4}(ID), ID\right)$ $\qquad ID = 0.73\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_8 - \text{CBH4}(ID), ID\right)$ $\qquad ID = 3.207\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_9 - \text{CBH4}(ID), ID\right)$ $\qquad ID = 4.455\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{10} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 5.382\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{11} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 6.111\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{12} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 6.741\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{13} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 7.294\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{14} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 7.784\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{15} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 8.222\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{16} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 8.621\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{17} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 8.982\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{18} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 9.297\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{19} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 9.578\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{20} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 9.849\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{21} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 10.122\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{22} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 10.351\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{23} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 10.521\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{24} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 10.653\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{25} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 10.761\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{26} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 10.856\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{27} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 10.941\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{28} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 11.019\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{29} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 11.091\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{30} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 11.158\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{34} - \text{CBH4}(ID), ID\right)$ $\qquad ID = 11.395\text{ cm}$

3 in from bridge $ID := 2.511\text{cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_9 - \text{CBH3}(ID), ID\right)$ $\qquad ID = 2.511\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{10} - \text{CBH3}(ID), ID\right)$ $\qquad ID = 4.005\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{11} - \text{CBH3}(ID), ID\right)$ $\qquad ID = 5.062\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{12} - \text{CBH3}(ID), ID\right)$ $\qquad ID = 5.874\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{13} - \text{CBH3}(ID), ID\right)$ $\qquad ID = 6.536\text{ cm}$

$ID := \text{root}\left(\text{WTh} - \text{Dr}_{14} - \text{CBH3}(ID), ID\right)$ $\qquad ID = 7.121\text{ cm}$

$$ID := root\left(WTh - Dr_{15} - CBH3(ID), ID\right) \qquad ID = 7.64\,cm$$
$$ID := root\left(WTh - Dr_{16} - CBH3(ID), ID\right) \qquad ID = 8.102\,cm$$
$$ID := root\left(WTh - Dr_{17} - CBH3(ID), ID\right) \qquad ID = 8.518\,cm$$
$$ID := root\left(WTh - Dr_{18} - CBH3(ID), ID\right) \qquad ID = 8.9\,cm$$
$$ID := root\left(WTh - Dr_{19} - CBH3(ID), ID\right) \qquad ID = 9.238\,cm$$
$$ID := root\left(WTh - Dr_{20} - CBH3(ID), ID\right) \qquad ID = 9.534\,cm$$
$$ID := root\left(WTh - Dr_{21} - CBH3(ID), ID\right) \qquad ID = 9.808\,cm$$
$$ID := root\left(WTh - Dr_{22} - CBH3(ID), ID\right) \qquad ID = 10.082\,cm$$
$$ID := root\left(WTh - Dr_{23} - CBH3(ID), ID\right) \qquad ID = 10.338\,cm$$
$$ID := root\left(WTh - Dr_{24} - CBH3(ID), ID\right) \qquad ID = 10.538\,cm$$
$$ID := root\left(WTh - Dr_{25} - CBH3(ID), ID\right) \qquad ID = 10.686\,cm$$
$$ID := root\left(WTh - Dr_{26} - CBH3(ID), ID\right) \qquad ID = 10.804\,cm$$
$$ID := root\left(WTh - Dr_{27} - CBH3(ID), ID\right) \qquad ID = 10.906\,cm$$
$$ID := root\left(WTh - Dr_{28} - CBH3(ID), ID\right) \qquad ID = 10.995\,cm$$
$$ID := root\left(WTh - Dr_{29} - CBH3(ID), ID\right) \qquad ID = 11.076\,cm$$
$$ID := root\left(WTh - Dr_{30} - CBH3(ID), ID\right) \qquad ID = 11.151\,cm$$
$$ID := root\left(WTh - Dr_{34} - CBH3(ID), ID\right) \qquad ID = 11.408\,cm$$

2 in from bridge $\qquad ID := 1.467\,cm$

$$Dist_7 - Dist_4 - 2in = 6.8\,in$$

$$ID := root\left(WTh - Dr_{10} - CBH2(ID), ID\right) \qquad ID = 1.467\,cm$$
$$ID := root\left(WTh - Dr_{11} - CBH2(ID), ID\right) \qquad ID = 3.491\,cm$$
$$ID := root\left(WTh - Dr_{12} - CBH2(ID), ID\right) \qquad ID = 4.692\,cm$$
$$ID := root\left(WTh - Dr_{13} - CBH2(ID), ID\right) \qquad ID = 5.601\,cm$$
$$ID := root\left(WTh - Dr_{14} - CBH2(ID), ID\right) \qquad ID = 6.315\,cm$$
$$ID := root\left(WTh - Dr_{15} - CBH2(ID), ID\right) \qquad ID = 6.934\,cm$$
$$ID := root\left(WTh - Dr_{16} - CBH2(ID), ID\right) \qquad ID = 7.482\,cm$$
$$ID := root\left(WTh - Dr_{17} - CBH2(ID), ID\right) \qquad ID = 7.97\,cm$$
$$ID := root\left(WTh - Dr_{18} - CBH2(ID), ID\right) \qquad ID = 8.407\,cm$$
$$ID := root\left(WTh - Dr_{19} - CBH2(ID), ID\right) \qquad ID = 8.806\,cm$$
$$ID := root\left(WTh - Dr_{20} - CBH2(ID), ID\right) \qquad ID = 9.168\,cm$$
$$ID := root\left(WTh - Dr_{21} - CBH2(ID), ID\right) \qquad ID = 9.484\,cm$$
$$ID := root\left(WTh - Dr_{22} - CBH2(ID), ID\right) \qquad ID = 9.766\,cm$$
$$ID := root\left(WTh - Dr_{23} - CBH2(ID), ID\right) \qquad ID = 10.039\,cm$$
$$ID := root\left(WTh - Dr_{24} - CBH2(ID), ID\right) \qquad ID = 10.313\,cm$$

$$ID := root\left(WTh - Dr_{25} - CBH2(ID), ID\right) \qquad ID = 10.542\,cm$$

$$ID := root\left(WTh - Dr_{26} - CBH2(ID), ID\right) \qquad ID = 10.712\,cm$$

$$ID := root\left(WTh - Dr_{27} - CBH2(ID), ID\right) \qquad ID = 10.844\,cm$$

$$ID := root\left(WTh - Dr_{28} - CBH2(ID), ID\right) \qquad ID = 10.952\,cm$$

$$ID := root\left(WTh - Dr_{29} - CBH2(ID), ID\right) \qquad ID = 11.047\,cm$$

$$ID := root\left(WTh - Dr_{30} - CBH2(ID), ID\right) \qquad ID = 11.132\,cm$$

$$ID := root\left(WTh - Dr_{34} - CBH2(ID), ID\right) \qquad ID = 11.414\,cm$$

1 in from bridge $\qquad ID := 2.838\,cm$

$$ID := root\left(WTh - Dr_{12} - CBH1(ID), ID\right) \qquad ID = 2.838\,cm$$

$$ID := root\left(WTh - Dr_{13} - CBH1(ID), ID\right) \qquad ID = 4.256\,cm$$

$$ID := root\left(WTh - Dr_{14} - CBH1(ID), ID\right) \qquad ID = 5.289\,cm$$

$$ID := root\left(WTh - Dr_{15} - CBH1(ID), ID\right) \qquad ID = 6.085\,cm$$

$$ID := root\left(WTh - Dr_{16} - CBH1(ID), ID\right) \qquad ID = 6.734\,cm$$

$$ID := root\left(WTh - Dr_{17} - CBH1(ID), ID\right) \qquad ID = 7.311\,cm$$

$$ID := root\left(WTh - Dr_{18} - CBH1(ID), ID\right) \qquad ID = 7.826\,cm$$

$$ID := root\left(WTh - Dr_{19} - CBH1(ID), ID\right) \qquad ID = 8.287\,cm$$

$$ID := root\left(WTh - Dr_{20} - CBH1(ID), ID\right) \qquad ID = 8.703\,cm$$

$$ID := root\left(WTh - Dr_{21} - CBH1(ID), ID\right) \qquad ID = 9.085\,cm$$

$$ID := root\left(WTh - Dr_{22} - CBH1(ID), ID\right) \qquad ID = 9.425\,cm$$

$$ID := root\left(WTh - Dr_{23} - CBH1(ID), ID\right) \qquad ID = 9.722\,cm$$

$$ID := root\left(WTh - Dr_{24} - CBH1(ID), ID\right) \qquad ID = 9.997\,cm$$

$$ID := root\left(WTh - Dr_{25} - CBH1(ID), ID\right) \qquad ID = 10.272\,cm$$

$$ID := root\left(WTh - Dr_{26} - CBH1(ID), ID\right) \qquad ID = 10.529\,cm$$

$$ID := root\left(WTh - Dr_{27} - CBH1(ID), ID\right) \qquad ID = 10.729\,cm$$

$$ID := root\left(WTh - Dr_{28} - CBH1(ID), ID\right) \qquad ID = 10.877\,cm$$

$$ID := root\left(WTh - Dr_{29} - CBH1(ID), ID\right) \qquad ID = 10.995\,cm$$

$$ID := root\left(WTh - Dr_{30} - CBH1(ID), ID\right) \qquad ID = 11.097\,cm$$

$$ID := root\left(WTh - Dr_{34} - CBH1(ID), ID\right) \qquad ID = 11.412\,cm$$

VII.D.   **Specifying positions of each drilling depth at Bridge Depth_Bridge**

At Bridge

$$\text{ID} := 1.96 \text{cm}$$

$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{13} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 1.96 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{14} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 3.752 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{15} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 4.928 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{16} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 5.819 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{17} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 6.518 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{18} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 7.128 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{19} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 7.67 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{20} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 8.156 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{21} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 8.592 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{22} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 8.991 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{23} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 9.354 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{24} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 9.671 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{25} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 9.955 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{26} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 10.229 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{27} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 10.504 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{28} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 10.733 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{29} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 10.903 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{30} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 11.035 \text{ cm}$
$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{34} - \text{B\_base(ID)}, \text{ID}\right)$	$\text{ID} = 11.402 \text{ cm}$

Finding where each line intersects the centerline (as measured from top of GuQin)

$$\text{Downslope} := \begin{pmatrix} \text{Dist}_4 - \text{Dist}_7 & \text{B\_base(0in)} \\ \text{Dist}_4 + 1\text{in} & \text{CBH1}(0{\cdot}\text{in}) \\ \text{Dist}_4 + 2\text{in} & \text{CBH2}(0\text{in}) \\ \text{Dist}_4 + 3\text{in} & \text{CBH3}(0\text{in}) \\ \text{Dist}_4 + 4\text{in} & \text{CBH4}(0\text{in}) \\ \text{Dist}_4 + 5\text{in} & \text{CBH5}(0\text{in}) \\ \text{Dist}_7 - 3.3\text{in} & \text{CHui1}(0\text{in}) \end{pmatrix}$$

$$\text{DownslopeConstant} := \text{lspline}\left(\text{Downslope}^{\langle 1 \rangle}, \text{Downslope}^{\langle 2 \rangle}\right)$$

$$\text{DS(X)} := \text{interp}\left(\text{DownslopeConstant}, \text{Downslope}^{\langle 1 \rangle}, \text{Downslope}^{\langle 2 \rangle}, X\right)$$

$$\text{DSD} := 43.27\,\text{cm}$$

$\text{DSD} := \text{root}\left(\text{DS(DSD)} + \text{Dr}_5 - \text{WTh}, \text{DSD}\right)$ $\text{DSD} = 22.02\,\text{cm}$ $\text{DSD} - \text{Dist}_7 = -3.431\,\text{in}$

$\text{DSD} := \text{root}\left(\text{DS(DSD)} + \text{Dr}_6 - \text{WTh}, \text{DSD}\right)$ $\text{DSD} = 20.234\,\text{cm}$ $\text{DSD} - \text{Dist}_7 = -4.134\,\text{in}$

$\text{DSD} := \text{root}\left(\text{DS(DSD)} + \text{Dr}_7 - \text{WTh}, \text{DSD}\right)$ $\text{DSD} = 18.447\,\text{cm}$ $\text{DSD} - \text{Dist}_7 = -4.837\,\text{in}$

$\text{DSD} := \text{root}\left(\text{DS(DSD)} + \text{Dr}_8 - \text{WTh}, \text{DSD}\right)$ $\text{DSD} = 16.664\,\text{cm}$ $\text{DSD} - \text{Dist}_7 = -5.539\,\text{in}$

$\text{DSD} := \text{root}\left(\text{DS(DSD)} + \text{Dr}_9 - \text{WTh}, \text{DSD}\right)$ $\text{DSD} = 14.876\,\text{cm}$ $\text{DSD} - \text{Dist}_7 = -6.243\,\text{in}$

$\text{DSD} := \text{root}\left(\text{DS(DSD)} + \text{Dr}_{10} - \text{WTh}, \text{DSD}\right)$ $\text{DSD} = 13.078\,\text{cm}$ $\text{DSD} - \text{Dist}_7 = -6.951\,\text{in}$

$\text{DSD} := \text{root}\left(\text{DS(DSD)} + \text{Dr}_{11} - \text{WTh}, \text{DSD}\right)$ $\text{DSD} = 11.322\,\text{cm}$ $\text{DSD} - \text{Dist}_7 = -7.642\,\text{in}$

$\text{DSD} := \text{root}\left(\text{DS(DSD)} + \text{Dr}_{12} - \text{WTh}, \text{DSD}\right)$ $\text{DSD} = 9.342\,\text{cm}$ $\text{DSD} - \text{Dist}_7 = -8.422\,\text{in}$

Bridge slot (like 36th line)

$$\text{ID} := 11.542\,\text{cm}$$

$$\text{ID} := \text{root}\left(\text{WTh} - \text{Dr}_{36} - \text{B\_base(ID)}, \text{ID}\right) \quad \text{ID} = 11.542\,\text{cm}$$

$$\text{B\_base(ID)} = -0.458\,\text{cm}$$

VIII. Calculating the Hui Positioning

$$\text{String\_length} := \text{Distance}(4, 12)$$

$\text{Dist\_Hui}_1 := \dfrac{1}{8} \cdot \text{String\_length} + \text{Dist}_4$ $\text{Dist\_Hui}_8 := \dfrac{3}{5} \cdot \text{String\_length} + \text{Dist}_4$

$\text{Dist\_Hui}_2 := \dfrac{1}{6} \cdot \text{String\_length} + \text{Dist}_4$ $\text{Dist\_Hui}_9 := \dfrac{4}{6} \cdot \text{String\_length} + \text{Dist}_4$

$\text{Dist\_Hui}_3 := \dfrac{1}{5} \cdot \text{String\_length} + \text{Dist}_4$ $\text{Dist\_Hui}_{10} := \dfrac{3}{4} \cdot \text{String\_length} + \text{Dist}_4$

$$\text{Dist\_Hui}_4 := \frac{2}{8} \cdot \text{String\_length} + \text{Dist}_4 \qquad \text{Dist\_Hui}_{11} := \frac{4}{5} \cdot \text{String\_length} + \text{Dist}_4$$

$$\text{Dist\_Hui}_5 := \frac{2}{6} \cdot \text{String\_length} + \text{Dist}_4 \qquad \text{Dist\_Hui}_{12} := \frac{5}{6} \cdot \text{String\_length} + \text{Dist}_4$$

$$\text{Dist\_Hui}_6 := \frac{2}{5} \cdot \text{String\_length} + \text{Dist}_4 \qquad \text{Dist\_Hui}_{13} := \frac{7}{8} \cdot \text{String\_length} + \text{Dist}_4$$

$$\text{Dist\_Hui}_7 := \frac{1}{2} \cdot \text{String\_length} + \text{Dist}_4$$

$$\text{Dist\_Hui} = \begin{pmatrix} 22.35 \\ 27.01 \\ 30.73 \\ 36.32 \\ 45.64 \\ 53.09 \\ 64.26 \\ 75.44 \\ 82.89 \\ 92.20 \\ 97.79 \\ 101.52 \\ 106.17 \end{pmatrix} \text{cm} \qquad \text{Dist\_Hui} = \begin{pmatrix} 7.2 \\ 8.7 \\ 9.9 \\ 11.7 \\ 14.7 \\ 17.1 \\ 20.7 \\ 24.3 \\ 26.7 \\ 29.7 \\ 31.5 \\ 32.7 \\ 34.2 \end{pmatrix} \text{cun}$$

IX.   Designing the Bottom Panel

IX.A.   Calculating the positions of the Pool and Pond

Lines numbered 2, 3, 4, 5, 9, 11, and 12 correspond to that of the Top Panel

"In the middle between the fourth and seventh hui markers, measure off four cun up and down, altogether equal to eight cun , which stand for the eight winds. This is the length for the dragon pool."

$$\text{Dist\_Mid\_dragonpool} := \frac{\left(\text{Dist\_Hui}_7 - \text{Dist\_Hui}_4 \right)}{2} + \text{Dist\_Hui}_4$$

$$\text{Dist\_Mid\_dragonpool} = 50.292 \, \text{cm}$$

$$4\text{cun} = 12.418 \, \text{cm}$$

"In the middle of the tenth and thirteenth hui positions, measure off two inches up and down, altogether equal to 4 cun , which represents the four seasons. This is the length of the phoenix pond.

$$\text{Dist\_Mid\_phoenixpond} := \frac{\left(\text{Dist\_Hui}_{13} - \text{Dist\_Hui}_{10}\right)}{2} + \text{Dist\_Hui}_{10}$$

$$\text{Dist\_Mid\_phoenixpond} = 99.187 \, \text{cm}$$

$$2\text{cun} = 6.209 \, \text{cm}$$

"Now from the axis measure off $4/_{10}$ cun for the width of the pool and of the pond, altogether $8/_{10}$ cun to the left and right."

$$0.4\text{cun} = 1.242 \, \text{cm}$$

$$0.8\text{cun} = 2.484 \, \text{cm}$$

IX.B.   Calculating the positions for the Wild Geese Feet holes

"At the center of the waist, away from the sides $3\text{-}4/_{10}$ cun on both left and right, draw lines to make squares $1/_2$ cun on a side for the holes for the wild geese feet."

$$0.35\text{cun} = 1.087 \, \text{cm}$$

$$0.5\text{cun} = 1.552 \, \text{cm}$$

IX.C.   Calculating the position of the Peg Pool

"From the third crosswise line, measure up and down $4/_{10}$ cun , altogether $8/_{10}$ cun . Now from the baseline measure out on left and right $2 \, 2/_{10}$ cun , altogether $4 \, 4/_{10}$ cun .

$$0.4\text{cun} = 1.242 \, \text{cm}$$

$$2.2\text{cun} = 6.83 \, \text{cm}$$

IX.D.   Determining the thickness of the Bottom Panel

Total Thickness	$0.6\text{cun} = 1.863 \, \text{cm}$
Demarcating line	$0.35\text{cun} = 1.087 \, \text{cm}$
Thickness of curves	$0.25\text{cun} = 0.776 \, \text{cm}$

X.   Calculating the dimensions of the accessories

X.A.   Bridge

Width   0.3cun = 0.931 cm

Set into face   0.3cun = 0.931 cm

Height above face   0.45cun = 1.397 cm

X.B.   Nut

Height   0.05cun = 0.155 cm

X.C.   Receiving Dew

Thickness   0.2cun = 0.621 cm

Width   0.4cun = 1.242 cm

X.D.   Peg Pool

Thickness   0.2cun = 0.621 cm

X.E.   Gums Supporter

Thickness   0.25cun = 0.776 cm
Breadth   2cun = 6.209 cm

X.F.   Wild Duck Feet

Square   1cun = 3.104 cm
Height   1.5cun = 4.657 cm

X.G.   Wild Geese Feet

Diameter   1cun = 3.104 cm
Height of round part   0.5cun = 1.552 cm
Square of post   0.5cun = 1.552 cm
Height of post   1.25cun = 3.881 cm
Distance embedded   0.45cun = 1.397 cm

XI.   Sound Posts

XI.A.   Pillar of Heaven

Round diameter $0.3\text{cun} = 0.931\text{ cm}$

$$\text{Position\_Pillar\_h} := \frac{\left(\text{Dist\_Hui}_4 - \text{Dist\_Hui}_3\right)}{2} + \text{Dist\_Hui}_3$$

$\text{Position\_Pillar\_h} = 10.8\text{ cun}$

XI.B.   Pillar of Earth

Square diameter $0.3\text{cun} = 0.931\text{ cm}$

$$\text{Position\_Pillar\_e} := \frac{\left(\text{Dist\_Hui}_8 - \text{Dist\_Hui}_7\right)}{2} + \text{Dist\_Hui}_7$$

$\text{Position\_Pillar\_e} = 22.5\text{ cun}$